### One-dimensional kinematics

- displacement: \( \Delta x = x_f - x_i \)
- average speed = total distance traveled / total time
- average velocity: \( v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \)
- instantaneous velocity: \( v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \)
- average acceleration: \( a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \)
- instantaneous acceleration: \( a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2} \)
- One-dimensional motion with constant acceleration:
  1. \( v = v_0 + at \)
  2. \( x = x_0 + \frac{1}{2}(v_0 + v)t \)
  3. \( x = x_0 + v_0t + \frac{1}{2}at^2 \)
  4. \( v^2 = v_0^2 + 2a(x - x_0) \)

- Free fall (positive direction for \( y \) taken to be upward)
  * \( x \to y \) and \( a \to -g \) in the above 4 equations of kinematics:
  1. \( v = v_0 - gt \)
  2. \( y = y_0 + \frac{1}{2}(v_0 + v)t \)
  3. \( y = y_0 + v_0t - \frac{1}{2}gt^2 \)
  4. \( v^2 = v_0^2 - 2g(y - y_0) \)

### Vectors

Vectors in 2-D

If a vector \( \vec{A} \) is written in component form as \( \vec{A} = A_i \hat{i} + A_j \hat{j} = (A_x, A_y) \) then:

- Getting magnitude and direction of \( \vec{A} \) from the components:
  \[ |\vec{A}| = \sqrt{A_x^2 + A_y^2} \quad \text{(magnitude of \( \vec{A} \))} \]
  \[ \theta = \begin{cases} \tan^{-1} \left( \frac{A_x}{A_y} \right), & \text{Quadrants I or IV} \\ \pm 180^\circ, & \text{Quadrants II or III} \end{cases} \quad \text{(direction of \( \vec{A} \))} \]
Getting components from magnitude and direction:
\[ A_x = |\vec{A}| \cos \theta \] (x component of \( \vec{A} \)) \[ A_y = |\vec{A}| \sin \theta \] (y component of \( \vec{A} \))
\[ \theta \text{ understood to be the angle that} \ \vec{A} \text{ makes with the positive } x \text{ axis} \]

Vectors in 3-D
If \( \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} = \{A_x, A_y, A_z\} \) then:

\[ |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \]

2-D Kinematics
- position vector: \( \vec{r}(t) = x(t) \hat{i} + y(t) \hat{j} = \langle x(t), y(t) \rangle \)
- average velocity: \( \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \left\langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t} \right\rangle \)
- instantaneous velocity: \( \vec{v} = \frac{d\vec{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle \)
- average acceleration: \( \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \left\langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t} \right\rangle \)
- instantaneous acceleration:
  - \( \vec{a} = \frac{d\vec{v}}{dt} = \left\langle \frac{dv_x}{dt}, \frac{dv_y}{dt} \right\rangle \)
  - \( \vec{a} = \frac{d^2 \vec{r}}{dt^2} = \left\langle \frac{d^2 x}{dt^2}, \frac{d^2 y}{dt^2} \right\rangle \)
- instantaneous speed \( \|\vec{v}\| \) (magnitude of the instantaneous velocity)

3-D kinematics
- position vector: \( \vec{r}(t) = x(t) \hat{i} + y(t) \hat{j} + z(t) \hat{k} = \langle x(t), y(t), z(t) \rangle \)
- average velocity: \( \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \left\langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t} \right\rangle \)
- instantaneous velocity: \( \vec{v} = \frac{d\vec{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle \)
- average acceleration: \( \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \left\langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t} \right\rangle \)
- instantaneous acceleration:
  - \( \vec{a} = \frac{d\vec{v}}{dt} = \left\langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \right\rangle \)
  - \( \vec{a} = \frac{d^2 \vec{r}}{dt^2} = \left\langle \frac{d^2 x}{dt^2}, \frac{d^2 y}{dt^2}, \frac{d^2 z}{dt^2} \right\rangle \)
- instantaneous speed \( \|\vec{v}\| \) (magnitude of the instantaneous velocity)
Projectile Motion

- $x$ direction (motion with constant velocity):
  - $a_x = 0$
  - $v_x = v_{0x}$
  - $x = x_0 + v_{0x}t$

- $y$ direction (free fall ... positive direction for $y$ taken to be upward):
  1. $v_y = v_{0y} - gt$
  2. $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$
  3. $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$
  4. $v_y^2 = v_{0y}^2 - 2g(y - y_0)$

Relative Motion

- $\ddot{v}_{PA} = \ddot{v}_{PB} + \ddot{v}_{BA}$

Newton's Laws of Motion

- Two broad categories of forces: contact forces (objects in contact with one another) and field forces (objects not in contact with one another). Gravity is the only field force we will deal with in this course.
- Weight: $w = mg$
- 1st law: $\ddot{v}$ is constant unless object (or system) experiences net external force.
- 2nd law:
  - $\sum F = m\ddot{a}$
  *implies three statements, in general: $\sum F_x = ma_x$, $\sum F_y = ma_y$, and $\sum F_z = ma_z$
  *for systems of objects, $\sum F_{ext} = m_{sys}a_{sys}$
- 3rd law: Whenever one object exerts a force on a second object, the second exerts a force on the first; these two forces are equal in magnitude and opposite in direction:
  - $\vec{F}_{12} = -\vec{F}_{21}$

Equilibrium

- An object is said to be “in equilibrium” (really in “translational” equilibrium) if:
  - $\sum F = 0 \Rightarrow \ddot{a} = 0$
  *really implies three separate requirements for translational equilibrium:
    1) $\sum F_x = 0 \Rightarrow a_x = 0$
    2) $\sum F_y = 0 \Rightarrow a_y = 0$
    3) $\sum F_z = 0 \Rightarrow a_z = 0$

Friction forces

- $f_s \leq \mu_s n$
- $f_k = \mu_k n$
**Circular Motion**

- 2nd law (centripetal direction): \[ (\sum F)_{\text{rad}} = ma_{\text{rad}} \]
- Radial (centripetal) acceleration: \[ a_{\text{rad}} = a_{\perp} = \frac{v^2}{r} \]
- If there is a tangential acceleration \( a_{||} \) also, then: \[ a_{||} = \frac{d|v|}{dt} \]