Detailed Answers

ANSWERS to MATH 1350 Extra Review Questions for Exam 4

1. For which value(s) of \( k \) can \( \frac{k}{1400} \) be written as a terminating decimal?

\[
\frac{k}{2 \cdot 7 \cdot 25 \cdot 2.5}
\]

any \( k \) that is a multiple of 7.

(Or the 7 in denominator can cancel)

2. In Room A there are 2 men and 5 women. In Room B there are 4 men and 3 women. In Room C there are 3 men and 3 women.

(a) What is the ratio of men in Room B to all the men? \( \frac{4}{9} \)

(b) What is the ratio of all the women to the women in Room C? \( \frac{11}{3} \)

3. Write \( \frac{1}{24} \) as a decimal. What is the 527th decimal in the expansion of \( \frac{1}{24} \)?

\[
\begin{array}{c|cccc}
\multicolumn{1}{c|}{24} & 1 & .0 & 4 & 16 \\
\hline
96 & \downarrow & & & \\
\hline
40 & & & & \\
24 & & & & \\
\hline
160 & & & & \\
144 & & & & \\
\hline
16 & & & & \\
\hline
0.0416 & & & & \\
\end{array}
\]

527th decimal is 6
4. Order the following from least to greatest:

\[ 0.45, \ 0.445, \ 0.45\overline{5}, \ 0.45\overline{4}, \ \frac{4}{9} \]

Compare:

\[
\begin{align*}
0.45000 & \quad 0.44500 \quad 0.45555 \\
0.45444 & \quad 0.45444 \\
\end{align*}
\]

\[ \text{least: } \frac{4}{9}, 0.445, 0.45, 0.45\overline{4}, 0.45\overline{4} \text{ greatest} \]

5. Write the following as a numeral:

\[ \text{five hundred two thousand forty and thirty-six hundred-thousandths} \]

\[ 502,040.00036 \]

6. Change \( 6.93\overline{2} \) to a rational number in the form \( \frac{a}{b} \), where \( a \) and \( b \) are integers (\( b \neq 0 \)).

\[
\begin{align*}
n &= 6.93\overline{2} \\
100n &= 693.\overline{2} \\
1000n &= 6932.\overline{2} \\
\frac{1000n}{900} &= \frac{6932.\overline{2} - 693.\overline{2}}{900} \\
n &= \frac{6239}{900}
\end{align*}
\]

7. Rewrite the following in scientific notation:

\[
\frac{(9 \cdot 10^7)(6 \cdot 10^4)}{(3 \cdot 10^6)} = \frac{54 \times 10^{11}}{3 \times 10^6} = 18 \times 10^5 = 1.8 \times 10^6
\]

8. Order the following decimals from greatest to least:

Compare:

\[
\begin{align*}
0.8 & \quad 0.8 & \quad 0.89 & \quad 0.89
\end{align*}
\]

\[ \text{least: } 0.8, 0.8, 0.89, 0.89 \text{ greatest} \]
9. Pyper invested $700 into an account paying simple interest at a rate of 8%/year. How much money (total) did she have after 10 months?

\[
A = P + I \\
= 700 + 700 \times (0.08)(\frac{10}{12}) \\
= \$746.67
\]

10. Change $-0.0\overline{7}$ to a rational number in the form \(\frac{a}{b}\), where \(a\) and \(b\) are integers \((b \neq 0)\).

\[
n = -0.0\overline{7} \\
100n = -7.\overline{7} \\
-10n = +.\overline{7} \\
90n = -7 \\
n = \frac{-7}{90}
\]

11. There are approximately 2 pounds of muscle for every 5 pounds of body weight. For a 90 pound person, approximately how much of the weight is muscle?

\[
\frac{2\text{lbs muscle}}{5\text{ lbs weight}} = \frac{x}{90\text{ lbs weight}} \\
2(90) = \frac{8x}{9} \\
\begin{align*}
2(90) &= \frac{8x}{9} \\
180 &= \frac{8x}{9} \\
x &= 36
\end{align*}
\]

12. What is the 6,543\textsuperscript{rd} decimal in the expansion of \(\frac{2}{21}\)?

\[
\frac{2}{21} = 0.095238 \text{ repeating} \\
6543 \div 6 = 1090 \text{ remainder } 3 \\
\therefore \text{ the } 6,543\text{rd} \text{ decimal is } 5
\]
13. If "decimals" in other bases work the same as in base ten, explain the meaning (in expanded form) of $204.3104_{\text{five}}$

$$2 (5^2) + 4 (5^1) + 3 (\frac{1}{5}) + 1 (\frac{1}{5^2}) + 4 (\frac{1}{5^3})$$

14. Change $2.\overline{4}$ to a rational number in the form $\frac{a}{b}$, where $a$ and $b$ are integers ($b \neq 0$).

\[
\begin{align*}
    n &= 2.\overline{4} \\
    10n &= 24.\overline{4} \\
    n - \frac{n}{9n} &= \frac{22}{9}
\end{align*}
\]

\[n = \frac{22}{9}\]

15. Mason can type 8 pages for every 6 pages that Jordan can type. If Jordan has typed 33 pages, how many pages has Mason typed?

\[
\begin{align*}
    \frac{\text{Mason}}{\text{Jordan}} &= \frac{8}{6} = \frac{x}{33} \\
    \frac{8(33)}{6} &= \frac{6x}{6} \\
    x &= 44
\end{align*}
\]

16. (a) Five hours is part of a day. If 5 hours were to be expressed as a decimal part of a day, explain whether or not it would be a terminating decimal.

\[\frac{5}{24} = \frac{5}{2^3 \cdot 3} \]

not terminating because there is a 3 in denominator and it can only have 2's and 5's if terminating.

(b) What whole number of hours (less than 24) could be expressed as terminating decimal parts of a day?

any hour that cancels the 3 in denominator.

3, 6, 9, 12, 15, 18, 21, 24
17. Does \( 0.\bar{5} = 1 \)? If yes, show why. If no, explain.

Yes!

\[
\begin{align*}
\text{n} &= 0.\bar{9} \\
10n &= 9.\bar{9} \\
\hline
q_n &= 0.9 \\
\text{n} &= 1
\end{align*}
\]

18. Five and a half years ago, Michael invested $10,000 in a retirement fund that grew at the rate of 10.82%/year compounded quarterly. What is his account worth today?

\[
\begin{array}{c}
PV = -10000 \\
\text{I} = 10.82 \\
p/y = 4 \rightarrow q/y = 4 \\
N = 4(5.5) \\
pmt = 0 \\
\end{array}
\]

\( FV = ? \)

\[\text{FV} = $17,989.33\]

19. Using only the digits 3 through 7 (each digit can be used only once), create the largest product possible for:

\[7.4 \times 6.5 = 48.1\]

20. Jamee takes out a short-term loan for $12,000. It is compounded weekly at 7.2% interest. She wants to pay it off using equal payments over the course of 5 years. How much interest will she end up paying on this loan?

\[
\begin{align*}
PV &= -12000 \\
FV &= 0 \\
\text{I} &= 7.2 \\
p/y = 52 \rightarrow q/y = 52 \\
N &= 52(5)
\end{align*}
\]

Find pmt 1st.

\[
pmt = \frac{54.99}{52} \times 52 - 12000 = \$2297.40
\]
21. Write $24.03405$ in expanded form.

\[ 2 \left(10^1\right) + 4 \left(10^0\right) + 3 \left(\frac{1}{10^1}\right) + 4 \left(\frac{1}{10^2}\right) + 5 \left(\frac{1}{10^5}\right) \]

22. What is 75% less than 320?

\[ 320 - 0.75(320) = 80 \]

23. A house was bought for $126,000 and is now worth $138,000. What is the percent increase in the value of the house?

\[
\begin{align*}
126,000 + p(126,000) &= 138,000 \\
p(126,000) &= 138,000 - 126,000 \\
p(126,000) &= \frac{12,000}{126,000} \\
p &= \frac{0.95238}{9.5\%}
\end{align*}
\]

24. Write the following as a numeral:

*eight hundred thirty-two million two thousand and six tenths*

\[ 832,002,000.6 \]
25. Use long division to rewrite $\frac{1}{13}$ as a decimal.

\[ \begin{array}{c|cccc}
\hline
13 & 1.00 & & & \\
- & 91 & & & \\
\hline
& 90 & & & \\
- & 78 & & & \\
\hline
& 120 & & & \\
- & 117 & & & \\
\hline
& 30 & & & \\
- & 26 & & & \\
\hline
& 40 & & & \\
- & 39 & & & \\
\hline
& 10 & & & \\
\hline
\end{array} \]

So, $0.076923 \ldots$

26. What is the 8546th digit in the decimal expansion of $\frac{1}{13}$?

- 6 digits in repeat
- $8546 \div 6 = 1424 \text{ remainder } 2$
- 2nd digit is 7

27. Write the following as numerals:

(a) Two and twenty-seven thousandths \[ 2.027 \]

(b) Two thousand and twenty-seven thousandths \[ 2000.027 \]

(c) Two thousand twenty and seven thousandths \[ 2020.007 \]

28. If the denominator of a fraction is 26, is it possible that the fraction could be written as a terminating decimal? Explain.

\[ \frac{2}{26} \]

Yes, if there is a prime factor of 13 in the numerator to cancel the one in the denominator. Ex: \[ \frac{13}{26} = \frac{1}{2} = 0.5 \]
29. Write 14.0479 in expanded place value form.

\[ 1 \left(10^1\right) + 4 \left(10^0\right) + 4 \left(\frac{1}{10^2}\right) + 7 \left(\frac{1}{10^3}\right) + 9 \left(\frac{1}{10^4}\right) \]

30. You want to buy a new car in 5 years so you decide to start a sinking fund that is compounded weekly at 2.8%. If you want to have $15,000 in five years,

(a) how much money should you put into the account each week?

\[ \text{pmt} = ? \]

\[ \text{\$ 53.76} \]

(b) how much interest will you end up earning on the account?

\[ 15000 - 53.76(5)(52) = \text{\$ 1022.40} \]

31. Write 305,209.074 in words.

three hundred five thousand two hundred nine and seventy four thousandths
32. A friend claims that every finite decimal is equal to some infinite decimal. Is the claim true? Explain.

Yes... could add repeating zeros or repeating nines.

Ex: $0.4 = 0.40000...$ since $0.4 + 0 + 0 + 0... = 0.40000... = 0.4$

Ex: $0.5 = 0.49$ since $0.4 + 0.9 = 0.4 + 1 = 0.5$

33. The mathematics final has 27 questions, each worth the same value. If Tanner needs to answer 86% of the questions correctly to earn a B in the course, how many questions can Tanner miss on the final?

Can miss 14% of them

$0.14(27) = 3.78$

So he can only miss 3

34. Your current salary is $37,000 and you receive a 2.5% raise. What is next year’s salary?

$37000 + 0.025(37000) = \$37,925$

35. Order the following decimals from greatest to least:

$1.4519, 1.4519, 1.4519, 1.4519$

Greatest: $1.451\bar{9}, 1.45\bar{1}9, 1.4\bar{5}19, 1.4\bar{5}19$ least
36. Rewrite the following in scientific notation:

\[
(6 \cdot 10^3)(5 \cdot 10^{-8}) \div (3 \cdot 10^{-7})
\]

\[
\frac{30 \cdot 10^{-5}}{3 \cdot 10^{-7}} = 10 \cdot 10^{-5} \cdot 10^7 = 10^3 = 1 \times 10^3
\]

37. Write \(\frac{1}{7}\) as a decimal.

\[
\begin{array}{c}
7 \div 10, \\
\underline{35} \\
50 \\
\underline{49} \\
1 \\
\underline{10} \\
7 \\
\underline{30} \\
28 \\
\underline{20} \\
14 \\
\underline{60} \\
56 \\
\underline{4} \equiv \text{repeat}
\end{array}
\]

\[0.142857142857142857\]

38. If the following letters correspond to \(N = \text{Natural numbers, I = Integers, Q = Rational numbers, R = real numbers and S = Irrational Numbers, complete the table by placing check marks in the appropriate columns.}\)

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>I</th>
<th>N</th>
<th>Q</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3+5i)</td>
<td></td>
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<td></td>
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<td></td>
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<tr>
<td>(-9)</td>
<td></td>
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<tr>
<td>(5/2)</td>
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<td>1.89</td>
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<tr>
<td>(4 \frac{1}{2})</td>
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<td>0</td>
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</tr>
</tbody>
</table>
39. Write 4.035 in the form \( \frac{\alpha}{\beta} \), where \( \alpha \) and \( \beta \) are integers and \( \beta \neq 0 \).

\[
\begin{align*}
\text{n} &= 4.035 \\
10n &= 40.35 \\
1000n &= 4035.35 \\
-10n &= -40.35 \\
990n &= 3995 \\
\frac{n}{990} &= 3.995 \\
\text{\( n = 3.995 \)}
\end{align*}
\]

40. Which of the following represents a terminating decimal:

\[
\begin{array}{c|c|c|c|c}
26 & 37 & 50 & 910 & 37 \\
65 & 625 & 260 & 1750 & 768 \\
\text{\( \text{Yes} \)} & \text{\( \text{Yes} \)} & \text{\( \text{No} \)} & \text{\( \text{Yes} \)} & \text{\( \text{No} \)}
\end{array}
\]

\[
\begin{align*}
\frac{2}{5} &= 0.4 \\
\frac{137}{54} &= \frac{2.5^2}{2.5^2 - 1.12} \\
\frac{2.5}{2.5^3} &= \frac{2.5^2 \times 13}{0.37} \\
\frac{37}{28.3} &= \text{not terminating}
\end{align*}
\]

41. Mark puts $4000 into an account that is compounded weekly at 3.6% annual interest rate. Five years later the rate goes up to 4.3% annual interest rate. If he leaves the money in this account, still compounding weekly, for another 3 years with no more deposits or withdrawals, how much money will he have at the end of this time?

\[
\begin{align*}
\text{PV} &= -4000 \\
\text{I} &= 3.6 \\
\text{N} &= 5(4) \\
\text{Pmt} &= 0 \\
\text{FV} &= 4785.02 \\
\text{I} &= 4.3 \\
\text{PV} &= -4785.02 \\
\text{N} &= 3(4) \\
\text{FV} &= ? \\
\$5440.12
\end{align*}
\]
42. A store marks up the price of an item 50%, and then puts it on sale at 25% off the marked up price. What percentage profit does the store actually make on the sale of the item?

\[ x + 0.50x = 1.5x \]

\[ 1.5x - 0.25(1.5x) = 1.5x - 0.375x = 1.125x \]

\[ \therefore 12.5\% \]

43. Cooper received an 8% raise last year. If his salary is now $18,181, what was his salary last year?

\[ x + 0.08x = 18181 \]

\[ 1.08x = 18181 \]

\[ \frac{1.08x}{1.08} = \frac{18181}{1.08} \]

\[ x = \$16,834.26 \]

44. What is the 2505th digit in the decimal expansion of \( \frac{3}{7} \)?

\[ 0.428571 \]

\[ \begin{array}{c}
7 \overline{3.0} \\
- 2.8 \\
\hline
0.2 \\
- 0.2 \\
\hline
0 \\
\end{array} \]

\[ \therefore 0.428571 \]

6 digit repetend

\[ 2505 \div 6 = 417 \text{ r } 3 \]

Remainder is 3

\[ 8 \] is the 3rd digit in the 3rd digit

45. Order the following numbers from least to greatest: 8.345, 8.345, 8.345

Least: 8.345, 8.345, 8.345

Greatest:

\[ 8.345000 \]

\[ 8.345555 \ldots \]

\[ 8.345454 \ldots \]
46. Compute the simple interest on a five month loan of $24,000 at a rate of 3.8% per year.

\[ I = Prt = 24000 \times 0.038 \times \left(\frac{5}{12}\right) = \$380 \]

47. If you add 9% of a number to itself, what percent of the result would you have to subtract to get back the original number?

\[ x + 0.09x = 1.09x \]
\[ 1.09x - p(1.09x) = x \]
\[ 1 - \frac{x}{1.09} = p \]
\[ p = 1 - \frac{1}{1.09} \]

48. Write the following as a numeral:

seven hundred nine million fifty two thousand six and three hundred eighty one millionths

\[ 709,052,006.000381 \]

49. Write the following in words: 4,035,245,008.0104

Four billion thirty five million two hundred forty five thousand eight and one hundred four ten-thousandths

50. The sale price of a cell phone is $350 and is advertised as being 15% off. What was the original price of the cell phone?

\[ x - 0.15x = 350 \]
\[ 0.85x = 350 \]
\[ x = \frac{350}{0.85} = \$411.76 \]

51. William kicks field goals one afternoon. He makes 32 field goals and his success-to-failure ratio is 8:3. How many field goals did he attempt?

\[ \frac{32}{x} = \frac{8}{3} \]
\[ \frac{8 \times 8}{8} = x \]
\[ x = \frac{64}{12} = \frac{32}{4} \times \frac{2}{4} = \frac{x}{44} \]

52. If there are 5 computers for every 12 students in a classroom, how many computers are there for the entire school of 252 students?

\[ \frac{5}{12} = \frac{x}{252} \]
\[ 5 \times \frac{252}{12} = \frac{12x}{12} \]
\[ x = 105 \]

53. If the denominator of a fraction 440, is it possible that the fraction could be written as a terminating decimal? Why or why not?

\[ 440 \times 10 \times 10 \times 10 \times 10 = 440 \times 10000 = 4400000 \]

Yes, if the numerator has a factor of 11. So it will cancel.

54. Change $-3.48$ to a rational number in the form of $\frac{a}{b}$, where $a$ and $b$ are integers ($b \neq 0$)

\[ n = -3.48 \]
\[ 100n = -348.48 \]
\[ -n = +\frac{3.48}{99} \]
\[ 99n = -345 \]