Review for Exam 2 by Dr. Poage

1. Is the set \( \{1, 2, 3\} \) closed under addition? Why or why not?

   \[ \text{No, Example: } 2 + 3 = 5, \ 5 \notin \{1, 2, 3\} \]

2. Mark takes a trip to another planet to visit his parents. On this planet the numeration system is in base three. His parents take him out to eat and the bill comes to 1221\(_{\text{three}}\).

   (a) If his parents hand the cashier 2000\(_{\text{three}}\), how much change do they receive (in base three)?

   \[ \begin{array}{c}
   2000_{\text{three}} \\
   -1221_{\text{three}} \\
   \hline
   2_{\text{three}}
   \end{array} \]

   (b) How much would this bill be in American money?

   \[ 1(3^3) + 2(3^2) + 1(3) = \$52 \]

3. Use rounding to estimate 82,345 + 5,826 + 2,535 by

   (a) round to the nearest thousand

   \[ 82,000 + 6,000 + 3,000 = 91,000 \]

   (b) round to the nearest hundred

   \[ 82,300 + 5,800 + 2,500 = 90,600 \]

   (c) round to the nearest ten

   \[ 82,350 + 5,830 + 2,540 = 90,720 \]

4. Write an expression in terms of \( d \) to show the amount of money, in cents, in a jar containing \( d \) dimes and some nickels and quarters, if there are three times as many nickels as dimes and twice as many quarters as nickels.

   \[ \begin{align*}
   n &= 3d \\
   a &= 2n = 2(3d) = 6d \\
   q &= 3n = 3(3d) = 9d \\
   d(10) + 3d(5) + 6d(25) &= 175d \text{ cents}
   \end{align*} \]

5. A child is asked to compute \( 7 + 2 + 3 + 8 + 11 \) and write \( 7 + 2 = 9 + 3 = 12 + 8 = 20 + 11 = 31 \). Noticing that the answer is correct, if you were the teacher how would you react?

   What you were thinking is correct, but what you wrote is incorrect

   \( 7 + 2 \) does not = 9 + 3, etc... 9 \( \neq \) 12, etc...

6. A student says that 1 is the identity for division. How do you respond?

   An identity is something that can be "operated" on either side of a \( \# \) to give you back that \( \# \). And while

   \[ a \div 1 = a, \ 1 \div a \neq a. \] Therefore it is not the identity for division.
7. Find the difference (in base six) for:
\[
\begin{array}{c}
5032_{six} - 2145_{six} \\
\hline
2443_{six}
\end{array}
\]

8. Set \( A \) contains the element 1. What other whole numbers must be in the set \( A \) for it to be closed under addition? Well, \( 1+1=2 \), \( 1+2=3 \), \( 1+3=4 \), …

9. Find the difference (in base 3) for:
\[
\begin{array}{c}
20010_{three} - 2022_{three} \\
\hline
10211_{three}
\end{array}
\]

10. Find the sum (in base 3) for:
\[
\begin{array}{c}
10122_{three} + 22101_{three} \\
\hline
110000_{three}
\end{array}
\]

11. Write the following statements in algebraic form:

(a) five times a number cubed
\[5x^3\]

(b) the sum of three consecutive integers if the greatest one is \( 5x \)
\[5x + 5x-1 + 5x-2 = 15x - 3\]

(c) seven less than two times a number
\[2x - 7\]

(d) the difference between the square root of eleven and eleven squared
\[\sqrt{11} - 11^2\]

(e) the product of three consecutive integers
\[x \cdot (x+1)(x+2) = x(x^2 + 2x + x + 2) = x^3 + 3x^2 + 2x\]

(f) the sum of four consecutive even integers
\[
\begin{align*}
(2x) + (2x+2) + (2x+4) + (2x+6) \\
= 8x + 12
\end{align*}
\]
12. If the domain is 2, 4, 6, 8, 10 and the range is 1, 3, 5, 7,

(a) draw a picture with arrows to demonstrate how this could be a function.

(b) draw a picture with arrows to demonstrate how this could NOT be a function.

13. Set $A$ is closed under addition and contains the numbers 3, 6, and 7. List six other elements that must be in $A$. Answers vary

Examples: 9, 11, 13, 12, 14, 20, 22, 21...

14. Find the sum (in base 5) for:
\[2_{\text{five}} + 3_{\text{five}}\]

\[\begin{array}{c}
1110_{\text{five}}
\end{array}\]

15. Draw pictures to find a solution to the following:
- two butterflies and a fish cost $14
- 1 butterfly, 1 fish, and 1 bird cost $17
- 1 bird and 1 butterfly cost $15

What is the cost of each?

Well, 1 butterfly & 1 bird (last box) cost $15

Using box 2, with that we see the fish must cost $2.

Using box 1, we see 2 butterflies must cost $12.

So $1 \Box = 4$, \( \Box \Box = 2 \), \( \Box \Box \Box = 9 \).
16. Write an example of base other than ten used in a real-life situation. How is it used?

17. Find the difference (in base eight) for:
\[ 6207_{eight} - 4534_{eight} \]
\[ \begin{array}{c}
\text{6207 eight} \\
- \text{4534 eight} \\
\hline
\text{1453 eight}
\end{array} \]

18. Explain whether the following sets are closed under addition:

(a) \( B = \{0, 1\} \)
No, \( 1 + 1 = 2, 2 \notin B \)

(b) \( T = \{0, 4, 8, 12, 16, \ldots\} \)
Yes, all multiples of 4.

(c) \( \{x \mid x \in W \text{ and } x > 5\} \)
\( \{5, 6, 7, 8, 9, 10, \ldots\} \)
Yes... sum could never be less than 10 \((5+5)\)...
20. When one attempts to divide by zero, what is the difference between “undefined” and “indeterminate”?

"Undefined" means there are no #s to multiply to another # to give you 0. Ex: \(a \div 0 = b \rightarrow \frac{a}{0} = b \rightarrow a = 0 \cdot b\)

"Indeterminate" means there is not a unique # to multiply to 0 to give you 0. Ex: \(0 \div 0 = b \rightarrow \frac{0}{0} = b \rightarrow 0 = 0 \cdot b\)

21. Multiply \(312_{\text{four}} \cdot 23_{\text{four}}\)

\[
\begin{array}{c}
\phantom{1}
2
2
0
2
\hline
+1
2
3
0
0
\hline
2
1
1
0
2
\text{four}
\end{array}
\]

22. A student claims that for all whole numbers, \(a \div a = 1\). How do you respond?

This is NOT true for all a. When \(a = 0\), \(\frac{0}{0} \neq 1\)

23. If the number 5 is removed from the set of whole numbers, A = \(\{0, 1, 2, 3, 4, 6, 7, 8, \ldots \}\), does the set remain closed under addition? EXPLAIN.

No, \(2 + 3 = 5\), \(5 \in A\)

24. A student says that 0 is the identity for both addition and subtraction. How do you respond?

It is for addition: \(a + 0 = 0 + a = a\).

It is not for subtraction: \(a - 0 = a\), but \(0 - a = -a\)

25. Divide \(24312_{\text{five}} \div 4_{\text{five}}\)

\[
\begin{array}{c|c}
\phantom{1}
3
3
1
3
\text{five}
\hline
\phantom{1}
2
2
\downarrow
\hline
2
3
1
2
\hline
-2
2
\downarrow
\hline
1
1
\downarrow
\hline
1
1
\downarrow
\hline
3
2
\text{five}
\end{array}
\]

\(3313_{\text{five}}\)
26. Which of the following are functions?

![Diagrams]

27. Write the sum of five consecutive even numbers if the middle one is $n$. Simplify your answer.

\[ n-4 + n-2 + n + n+2 + n+4 = 5n \]

28. Divide

\[ 1423 \div 3 \]

\[ \underline{1423} \]
\[ - \underline{123} \]
\[ \underline{190} \]
\[ - \underline{190} \]
\[ \underline{23} \]
\[ - \underline{23} \]
\[ 0 \]

\[ 325 \]

29. Can 0 be the identity for multiplication? Explain why or why not.

No, $a \cdot 0 = 0$, the identity gives you back the original # ... thus 1 is the identity for multiplication since $a \cdot 1 = a = 1 \cdot a$

30. A student says that 1 is the identity for division. How do you respond?

No, the operation has to work on both sides and while $a \div 1 = a$

$1 \div a \neq a$
31. Use LATTICE multiplication to multiply \(434_{five} \cdot 24_{five}\)

\[
\begin{array}{ccc}
4 & 3 & 4 \\
\hline
2 & 3 & 1 \\
3 & 1 & 3 \\
\end{array}
\]

\[
= 23131_{five}
\]

32. Multiply \(768_{nine} \cdot 57_{nine}\)

\[
\begin{array}{c}
\times \ 57_{nine} \\
\hline
6032 \\
42740 \\
\hline
48772_{nine}
\end{array}
\]

33. Divide \(1021_{three} \div 2_{three}\)

\[
\frac{122}{2_{three} \overline{|1021_{three}}} \\
- \frac{12}{11} \\
- \frac{11}{0}
\]

\[
= 122_{three}
\]

34. A health club charges an initiation fee of $200, which gives 1 month of free membership, and then charges $55 per month. Find the equation for \(C(x)\), the total cost of membership in the club for \(x\) months.

\[
C(x) = 200 + 55(x-1)
\]

35. Write an algebraic equation for the pay, \(P\), for \(t\) hr if you are paid $12 for the first hour and $8 for each additional hour.

\[
P = 12 + 8(t-1)
\]
36. Does the associative property hold for division? If yes, explain. If no, give a counterexample.

\[ \text{No, } a \div (b \div c) \neq (a \div b) \div c \]

Ex: \[ 8 \div (4 \div 2) \neq (8 \div 4) \div 2 \]
\[ 8 \div 2 \neq 2 \div 2 \]
\[ 4 \neq 1 \]

37. Multiply \(3024_{\text{five}} \cdot 43_{\text{five}}\).

\[
\begin{array}{c}
3024_{\text{five}} \\
\times 43_{\text{five}} \\
\hline
14132_{\text{five}} \\
+22110_{\text{five}} \\
\hline
241242_{\text{five}} \\
\end{array}
\]

38. A 10 ft board is to be cut into three pieces, two equal length ones and the third 3 in. shorter than each of the other two. If the cutting does not result in any length being lost, how long are the pieces? Set up the equation using only ONE variable.

\[ x + x + (x-3) = 120 \]
\[ 3x = 123 \]
\[ x = 41 \]

39. Divide \(3763_{\text{eight}} \div 5_{\text{eight}}\).

\[
\begin{array}{c}
3763_{\text{eight}} \\
\div 5_{\text{eight}} \\
\hline
627_{\text{eight}} \\
\hline
-120 \\
\hline
-48 \\
\hline
-48 \\
\hline
\end{array}
\]

40. In a college, there are 13 times as many students as professors. If together the students and professors numbers 28,000, how many students are there in the college? (write the equations and solve the problem)

\[ S = 13P \]
\[ S + P = 28000 \]
\[ 13P + P = 28000 \]
\[ \frac{14P}{14} = 28000 \]
\[ P = 2000 \]
\[ S(13)(2000) = 26,000 \]
41. Write the sum of five consecutive even numbers if the middle one is $n$. Simplify your answer.

$$n - 4 + n - 2 + n + n + 2 + n + 4 = 5n$$

42. Estimate $5412 + 3308 + 1202$ by using Two-column Front-end estimation.

$$5000 + 3000 + 1000 = 9000$$

$$900 + 300 + 100 = 900$$

$$\frac{9900}{9900}$$

43. What does the following expression equal right now? By placing parantheses in different places, turn the following expression into 5 different values.

1) $(12 - 4) \div 2 + 6 \cdot 4 \div 4 = 10$

2) $12 - 4 \div 2 + 6 \cdot (4 \div 4) = 16$

3) $12 - 4 \div (2 + 6) \cdot 4 \div 4 = 11.5$

4) $(12 - 4) \div (2 + 6) \cdot 4 \div 4 = 1$

5) $12 - 4 \div (2 + 6 - 4) \div 4 = \frac{33}{16}$

44. What is the difference between mental computation and estimation?

- Estimation is an approximation.
- Mental computation is finding the exact value without the aid of a calculator or computer.

45. Set $B$ is closed under multiplication and contains the numbers 4, 7, and 9. List at least six other elements that must be in $B$. Answers vary.

Examples: 16, 49, 81, 28, 63, 36, ...

46. A student claims that $0 \div 0 = 1$. How do you respond? Explain.

No, $0 \div 0 = 1$ implies $\frac{0}{0} = 1 \implies 0 = 0 \cdot 1$ where 1 is a unique number... but really any number then could make this true... $0 \div 0$ is indeterminate.

47. If Mason has twice as many CDs as Cooper and Tanner has 3 times as many as Mason, write an algebraic expression for the number of CDs each has in terms of one variable ($n$). (State what $n$ is)

Let $n = \#$ of CDs Cooper has. Then

- Mason = $2n$
- Cooper = $n$
- Tanner = $6n$
48. Mike has 3 times as many baseball cards as Jordan, who has twice as many cards as Paige. Together, the three children have 999 cards. Set up an equation in one variable, \( n \), and find how many cards each child has. (state what \( n \) is)

Let \( n \) = # cards Paige has, then

\[
\begin{align*}
\text{Paige} &= 111 \\
\text{Jordan} &= 222 \\
\text{Mike} &= 666
\end{align*}
\]

\[
\begin{align*}
n + 2n + 6n &= 999 \\
9n &= 999 \\
n &= 111
\end{align*}
\]

49. How would you use compatible numbers to find \( 20 \times 6 \times 5 \times 9 \)

\[
\begin{align*}
100 \times 54 &= 5400
\end{align*}
\]

50. \( 75_{\text{eight}} \times 6_{\text{eight}} \)

\[
\begin{align*}
5_{\text{eight}} &\quad 3_{\text{eight}} \\
\times &\quad 6_{\text{eight}} \\
\hline
55_{\text{eight}}
\end{align*}
\]

51. Use LATTICE addition to add \( 562_{\text{seven}} + 694_{\text{seven}} \)

\[
\begin{array}{c}
5_{\text{seven}} \\
5_{\text{seven}} \\
\hline
1_2_0_{\text{seven}}
\end{array}
\]

\[
\begin{array}{c}
4_1_6_{\text{seven}} \\
\hline
1_6_1_6_{\text{seven}}
\end{array}
\]

52. Ten years from now Tanner's age will be 3 times his present age. Find Tanner's age now. (set up the equation using one variable and solve)

\[
T + 10 = 3T \\
10 = 2T \\
5 = T
\]
53. $210_{\text{three}} \times 2_{\text{three}}$

\[
\begin{array}{c}
\times \\
\hline
1120_{\text{three}}
\end{array}
\]

54. Explain how WHERE you put parentheses in the following problem could completely change the answer: $9 - 3 \div 3 + 2 \cdot 4$

\[
\begin{align*}
\text{if: } (9 - 3) \div 3 + 2 \cdot 4 &= 10 \\
\text{(no parenth.) if: } 9 - 3 \div 3 + 2 \cdot 4 &= 16
\end{align*}
\]

\[
\begin{align*}
\text{if: } 9 - 3 \div (3 + 2) \cdot 4 &= \frac{23}{5} \\
\text{if: } 9 - (3 \div 3 + 2) \cdot 4 &= 0 \\
\text{if: } (9 - 3 \div 3 + 2) \cdot 4 &= 40
\end{align*}
\]

55. $53_{\text{six}} \times 14_{\text{six}}$

\[
\begin{array}{c}
\times \\
\hline
1310_{\text{six}}
\end{array}
\]

56. $10221_{\text{four}} \div 3_{\text{four}}$

\[
\begin{array}{c}
1203_{\text{four}}
\end{array}
\]

57. $342_{\text{five}} \times 11_{\text{five}}$

\[
\begin{array}{c}
\times \\
\hline
4312_{\text{five}}
\end{array}
\]
58. (a) What is the fewest number of quarters, nickels, and pennies you can receive in a fair exchange for two quarters, nine nickels, and eight pennies?

4 Quarters and 3 pennies.

50 quarters, 0 nickels, 3 pennies.

(b) How could you use the approach in part (a) to write 73 in base five?

7 nickels and 3 pennies
Trade 5 nickels for 1 quarter, so now
1 quarter, 2 nickels, and 3 pennies
or 123 five

59. $10011_{two} \times 10_{two}$

60. $34_{five} \cdot 22_{five}$

61. $21403_{five} \div 2_{five}$
62. Which properties are illustrated for each of the following?
   (a) \(4 \cdot (5 \cdot 6) = (4 \cdot 5) \cdot 6\) \(\text{associative}\)
   (b) \(2(6 + 9) = 2 \cdot 6 + 2 \cdot 9\) \(\text{distributive}\)
   (c) \(0 + 14 = 14\) \(\text{identity}\)

63. \(52_{\text{six}} \times 34_{\text{six}}\)

\[
\begin{array}{c}
52_{\text{six}} \\
\times 34_{\text{six}} \\
\hline
332 \\
+2400 \\
\hline
3132_{\text{six}} \\
\end{array}
\]

64. \(353_{\text{seven}} \div 5_{\text{seven}}\)

\[
\begin{array}{c}
52 \\
5_{\text{seven}} \backslash 353_{\text{seven}} \\
\hline
-34 \\
\hline
13 \\
-13 \\
\hline
0 \\
\end{array}
\]

65. Find the numeral to put in the blank to make each equation true.
   (a) \(3423_{\text{five}} - \underline{\hspace{2cm}} = 2131_{\text{five}}\)
   \(\text{Same as } 3423_{\text{five}} - 2131_{\text{five}} \rightarrow \frac{3423_{\text{five}}}{1242_{\text{five}}}\)
   (b) \(11011_{\text{two}} + \underline{\hspace{2cm}} = 100000_{\text{two}}\)
   \(\text{Same as } 100000_{\text{two}} - 11011_{\text{two}} \rightarrow 101_{\text{two}}\)

66. Multiply: \(4T8_{\text{twelve}} \cdot 2E_{\text{twelve}}\)

\[
\begin{array}{c}
4T8_{\text{twelve}} \\
\times 2E_{\text{twelve}} \\
\hline
4'9'71 \\
+9'9'40 \\
\hline
1'231'4_{\text{twelve}} \\
\end{array}
\]
67. Divide \(21403_{\text{five}} \div \text{2}_{\text{five}}\) \[\begin{array}{c}
21403_{\text{five}} \\
\underline{\text{2} \times \text{21403}_{\text{five}}}
\end{array}\]
\[\begin{array}{c}
-10424_{\text{five}} \\
\underline{\text{014}_{\text{five}}}
\end{array}\]
\[\begin{array}{c}
-13\text{v} \\
\underline{\text{19}_{\text{five}}}
\end{array}\]
\[= 10424_{\text{five}}\]

68. Mika takes a trip to the moon where they have a base five number system. While on the moon, Mika buys a shirt for \(132_{\text{five}}\), a pair of shoes for \(343_{\text{five}}\), and a jacket for \(1233_{\text{five}}\).

(a) How much Moonian money does Mika owe?

\[\begin{array}{c}
2_{\text{five}} \\
\underline{\text{132}_{\text{five}}}
\end{array}\]
\[\begin{array}{c}
\text{3}_{\text{five}} \\
\underline{\text{343}_{\text{five}}}
\end{array}\]
\[\begin{array}{c}
\text{1}_{\text{five}} \\
\underline{\text{1233}_{\text{five}}}
\end{array}\]
\[\text{2313}_{\text{five}}\]

(b) How much does this little shopping spree equate to in American money?

Convert \(2313_{\text{five}}\) to base 10.

\[2(5^3) + 3(5^2) + 1(5^1) + 3(5^0) = 333\]

(c) If Mika hands the clerk \(10000_{\text{five}}\) in Moonian bills, how much Moonian money should Mika get back from the clerk?

\[\begin{array}{c}
\text{10000}_{\text{five}} \\
\underline{\text{2313}_{\text{five}}}
\end{array}\]
\[\text{2132}_{\text{five}}\]

69. How could you use money to explain how to calculate \(48 \cdot 25\)?

48 Quarters ... Since 4 Quarters are in a dollar (or 100 cents)

then \(48 ÷ 4 = 12\) dollars or \(12(100) = 1200\)

70. Use LATTICE multiplication to find \(123_{\text{four}} \cdot 32_{\text{four}}\) (show all work)

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\hline
0 & 3 & 2 & 1 & 1 & 2 & 3 \\
0 & 2 & 1 & 0 & 1 & 2 & 0 \\
1 & 0 & 2 & 0 & 2 & 2 & 2 \\
\end{array}
\]
\[= 11322_{\text{four}}\]