

Chapter L

Temperature and Heat

Temperature Scales

Fahrenheit and Celsius Scales

Two commonly used temperature scales are Fahrenheit and Celsius. In the Fahrenheit scale the temperature of the freezing point of water (at one atmosphere) is 32 °F and the boiling point is 212 °F. For Celsius these two temperatures are 0 °C and 100 °C. It is straightforward to convert between the two. Consider a graph of T_F vs. T_C . We insist that these two scales are linearly related; this implies that the graph is a line. The slope of the line is

$$\text{slope} = \frac{\Delta T_F}{\Delta T_C} = \frac{212-32}{100-0} = \frac{9}{5}.$$

The intercept is 32. It follows that the conversion is

$$T_F = \frac{9}{5} T_C + 32.$$

Note that for temperature difference we get

$$\Delta T_F = \frac{9}{5} \Delta T_C.$$

Constant Volume Gas Thermometers

A thermometer based on the expansion of gas at constant volume was introduced. It was seen that a plot of pressure vs. temperature for different gases gave lines that at zero pressure intercepted the temperature axis at a very cold negative temperature. What was observed is that for different gases we got different lines but all gases intercepted the temperature axis at the same very cold temperature.

This was the first hint of a coldest temperature that we now call absolute zero. It should be mentioned that absolute zero is much more fundamental than just a property of cold gases. It is a very fundamental value and the coldest temperature in *all* thermal experiments. In the Celsius scale the value of absolute zero is

$$T_{\text{absolute zero}} = -273.15^\circ\text{C}.$$

Absolute Temperature and Kelvin

An absolute temperature scale is one that is shifted to make absolute zero, zero in that scale. The absolute scale associated with Celsius is called the Kelvin scale. The conversion between Celsius and Kelvin is

$$T_K = T_C + 273.15.$$

Temperatures in Kelvin are given as K and not °K. We usually will take the above number to be just 273. Note that temperature differences in Kelvin are the same as in Celsius. We will give these differences in C°; that is, °C is a temperature and C° is a temperature difference.

Thermal Expansion

Linear Expansion

When the temperature of a body increases it expands. Take L_0 to be the length of a rod at temperature T_0 . If the temperature increases to $T_0 + \Delta T$ then its length increases to $L_0 + \Delta L$. How does the expansion ΔL depend on the temperature difference ΔT ? A rod of twice the length $2L_0$ made of the same material will clearly expand by twice as much as the original rod, assuming that the ΔT is the same. This implies that there is a proportionality

$$\Delta L \propto L_0.$$

In addition there is another proportionality. For small temperature changes the expansion is proportional to it

$$\Delta L \propto \Delta T.$$

For larger temperature changes this proportionality is only approximately true.

Combining these proportionalities we get $\Delta L \propto L_0 \Delta T$. We now introduce a constant of proportionality α , the coefficient of linear thermal expansion. We get

$$\Delta L = \alpha L_0 \Delta T.$$

What sort of constant is α ? It is not some fundamental constant that we measure once. α is different for different materials. The exact definition of this constant is for small ΔT .

$$\alpha = \frac{dL}{L dT} = \frac{d}{dT} \ln L$$

Thermal Expansion is Uniform

Suppose there is a sheet of some metal with a hole in it. When the metal expands what happens to the size of the hole? Thermal expansion is uniform. The distances between any two points on an object expand at the same rate. It follows that the hole expands at the same rate as everything else. To get an intuitive model for this do not view it as similar to expanding dough; a hole in expanding dough gets smaller. Instead think of a piece of paper on a photocopier and choose to enlarge the image; the distance between any two points increases at the same rate.

Volume Expansion

For volume we similarly define V_0 as the volume at temperature T_0 and $V_0 + \Delta V$ as the volume at $T_0 + \Delta T$. We can introduce β as the coefficient of volume expansion. We get the expression

$$\Delta V = \beta V_0 \Delta T.$$

We can similarly give the exact definition of β as

$$\beta = \frac{dV}{V dT} = \frac{d}{dT} \ln V.$$

For a fluid (a liquid or gas) there is volume expansion but its linear expansion is not uniform, because it doesn't maintain its shape. Thus for a fluid β is defined but α is not. The linear expansion of a solid determines its volume expansion. β is related to α . We will next show that for a solid

$$\beta = 3\alpha.$$

Suppose one has a scale model of an object. If it is $1/2$ the height then it is $1/8$ the volume. For any reference distance in an object the volume is proportional to that distance cubed, $V \propto L^3$. This can be written as

$$\frac{V}{V_0} = \left(\frac{L}{L_0}\right)^3 \text{ or } V = \kappa L^3 \text{ where } \kappa = \frac{V_0}{L_0^3}.$$

κ is a constant that doesn't change as the object expands.

$$\beta = \frac{d}{dT} \ln V = \frac{d}{dT} (\kappa L^3) = \frac{d}{dT} \ln \kappa + 3 \frac{d}{dT} \ln L = 0 + 3 \alpha$$

This proves the result.

Heat

Heat is thermal energy that flows from hot to cold, or more precisely, from higher temperature to lower temperature. It is an essential point that heat is thermal energy that moves. The static notion of thermal energy is a quite different thing which we will define later as *internal energy*. We will use Q to denote heat. We will choose the convention that Q is the heat *added* to a thermodynamic system. When heat is removed from something we take Q to be negative.

What is the effect of heat on a system? Suppose you add heat to a pot of water. The heat will increase the temperature of the water, usually. But if the water is at the boiling point the heat doesn't change the temperature; it changes the phase. Thus, when heat is added to a system it can either change the temperature of the system or change its phase.

Temperature Change - Specific Heat

To raise the temperature of a fixed by some amount ΔT requires heat that is roughly proportional to the temperature change $Q \propto \Delta T$. This is precise as the ΔT becomes small. Moreover, to raise a substance by a fixed temperature requires an amount of heat proportional to m , the mass of the substance, $Q \propto m$. We can combine these proportionalities and get

$$Q = m c \Delta T.$$

c , the constant of proportionality is called the specific heat; it is a property of a material. Generally, it varies somewhat with the temperature and pressure of the substance, but we will typically neglect this change as small.

The specific heat c is a property of a material. The heat capacity C is a property of an object.

$$Q = C \Delta T$$

For example, a thermos has a heat capacity; the glass in a thermos has a specific heat. If an object is made of one material then $C = m c$. If it is made of several then $C = m_1 c_1 + m_2 c_2 + \dots$.

Phase Change - Latent Heat

The phase change between solids and liquids is called fusion. The liquid-gas transition is called vaporization. At the temperature of a phase transition the latent heat is the amount of heat per mass needed to change the phase.

$$Q = \pm m L$$