

# Problems - Chapter N

## Physics 2425 - Dr. Terry Honan

### ■ Problem N.1

What is the change in entropy of 3 kg of water initially at 20 °C when it is cooled and frozen resulting in ice at 0 °C?

#### Solution to N.1

This is a two step problem.  $\Delta S_1$  is the change in entropy when the water is brought from 20 °C to 0 °C and  $\Delta S_2$  is the change when the water at 0 °C freezes to 0 °C. When water freezes to ice heat is removed so  $Q = -mL_f$ . The sign of the entropy of the water to water case follows from the values of the temperatures. The temperature must be in Kelvin in all cases here.

$$\begin{aligned}\Delta S &= \Delta S_1 + \Delta S_2 = mc \ln \frac{T_f}{T_i} + \frac{Q}{T} = mc_{\text{water}} \ln \frac{T_f}{T_i} + \frac{-mL_f}{T} \\ &= 3 \times 4186 \ln \frac{273}{293} + \frac{-3 \times 3.33 \times 10^5}{273} = -887.9 - 3659.3 = -4550 \text{ J/K}\end{aligned}$$

### ■ Problem N.2

What is the total change in entropy when 3000 J of heat flows into a house at 20 °C from the warm exterior at 40 °C?

#### Solution to N.2

The entropy of the exterior decreases, since heat flows out of it, and the entropy of the interior increases since heat flows into it.

$$\begin{aligned}\Delta S_{\text{tot}} &= \Delta S_{\text{outside}} + \Delta S_{\text{inside}} = \frac{-Q}{T_{\text{outside}}} + \frac{Q}{T_{\text{inside}}} = \frac{-3000}{313} + \frac{3000}{293} \\ &= -9.585 + 10.239 = 0.654 \text{ J/K}\end{aligned}$$

The second law of thermodynamics says that the total change in entropy cannot be negative for any thermally isolated system. A system is thermally isolated if no heat flow into or out of it.

### ■ Problem N.3

What is the total change in entropy when a 20 g ice cube at 0 °C is dropped into the Gulf of Mexico, which is at a temperature 15 °C? It is *quite* safe to assume that the temperature change of the Gulf is negligible.

#### Solution to N.3

The heat that flows into the ice cube comes from the Gulf. This heat  $Q$  first melts the ice and then raises its temperature from  $0^\circ\text{C}$  to  $15^\circ\text{C}$ .

$$Q = mL_f + mc\Delta T = 0.02 \times 3.33 \times 10^5 + 0.02 \times 4186 \times 15 = 6660.00 + 1255.80 = 7915.80$$

Since this heat flows out of the Gulf, which is at constant temperature, its entropy change is

$$\Delta S_{\text{Gulf}} = \frac{-Q}{T} = \frac{-7915.80}{288} = -27.485 \text{ J/K}$$

The entropy change of the ice involves two terms, the first to melt it and the second to raise its temperature.

$$\begin{aligned} \Delta S_{\text{ice}} &= \Delta S_1 + \Delta S_2 = \frac{+mL_f}{T} + mc \ln \frac{T_f}{T_i} \\ &= \frac{0.02 \times 3.33 \times 10^5}{273} + 0.02 \times 4186 \ln \frac{288}{273} = 24.396 + 4.478 = 28.874 \text{ J/K} \end{aligned}$$

The total change is the sum of the two.

$$\Delta S_{\text{tot}} = \Delta S_{\text{Gulf}} + \Delta S_{\text{ice}} = 1.39 \text{ J/K}$$

#### ■ Problem N.4

A heat engine absorbs 6000 J of heat from a hot reservoir and expels 4100 J to a cold reservoir at  $30^\circ\text{C}$ .

- What is the efficiency of this engine?
- What is the work done by the engine?
- What is the smallest temperature the hot reservoir could have?

#### Solution to N.4

$$(a) e = 1 - \frac{Q_C}{Q_H} = 1 - \frac{4100}{6000} = 0.31667 = 31.7\%$$

$$(b) Q_H = Q_C + W \implies W = Q_H - Q_C = 6000 - 4100 = 1900 \text{ J}$$

(c) The efficiency has an upper limit of  $e_{\text{max}} = 1 - \frac{T_C}{T_H}$  where  $T$  must be in Kelvin.

$$e \leq e_{\text{max}} = 1 - \frac{T_C}{T_H} \implies T_H = \frac{T_C}{1-e} = \frac{303}{1-0.31667} = 443.4 \text{ K} = 170.4^\circ\text{C}$$

#### ■ Problem N.5

A Carnot engine that operates between a 500 K hot reservoir and a 300 K cold reservoir has a power output of 6000 W. What is the efficiency of this heat engine? At what rate does this engine absorb heat?

**Solution to N.5**

A Carnot engine is a theoretical heat engine of maximum efficiency.

$$e = e_c = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{500} = 0.40 = 40 \%$$

The power output is the work per time.

$$e = \frac{W}{Q_H} = \frac{W/t}{Q_H/t} = \frac{\mathcal{P}}{Q_H/t} \implies Q_H/t = \frac{\mathcal{P}}{e} = \frac{6000}{0.40} = 15\,000 \text{ W}$$