

Problems - Chapter A

Physics 2425 - Dr. Terry Honan

■ Problem A.1

Suppose that x is a length, t is a time and a is an acceleration. To get a general expression for x as a function of both t and a , we choose the general form:

$$x = \kappa a^m t^n$$

where κ is a dimensionless constant. For this expression to be dimensionally correct what must m and n be?

■ Problem A.2

Three fundamental constants G , c and \hbar have dimensions:

$$[G] = \frac{\text{L}^3}{\text{M}\cdot\text{T}^2}, [c] = \frac{\text{L}}{\text{T}} \text{ and } [\hbar] = \frac{\text{M}\cdot\text{L}^2}{\text{T}}.$$

(a) What must m , n and p be to make L_0 a length, when

$$L_0 = G^m \cdot c^n \cdot \hbar^p.$$

(b) \hbar , called "h bar", is a rescaled version of Planck's constant h ; it is usually also called Planck's constant.

$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{ J}\cdot\text{s}$$

Using this and the values of G and c given in the book evaluate L_0 .

Comment: G is Newton's Gravitational constant, c is the speed of light which plays a central role in Relativity and \hbar is called *Planck's constant*, which appears in Quantum theory. L_0 is known as the *Planck length*; it is the very small distance that will set the scale of some ultimate theory of *Quantum Gravity*. Quantum Gravity is some yet undiscovered theory that consistently combines Gravity, Relativity and Quantum theory.

■ Problem A.3

(a) The period (time) for small oscillations of a simple pendulum T is given by:

$$T = 2\pi \sqrt{\frac{L}{g}},$$

where L is the length of the pendulum and g is the acceleration due to gravity. Verify that this expression is dimensionally correct.

(b) The velocity as a function of time for small oscillations of a pendulum is given by:

$$v = A \sin(\omega t + \phi),$$

where t is time. A , ω and ϕ are constants, the amplitude, angular frequency and phase. What are the dimensions and SI units of these constants?

■ Problem A.4

Which of the following equations are dimensionally correct?

(a) $v^2 - v_0^2 = 2 a x$ (b) $x = v_0 t + \frac{1}{3} a t^3$ (c) $x = \frac{1}{2} (v + v_0) t^2$

v and v_0 are velocities, a is acceleration, t is time and x is a distance.

■ Problem A.5

Newton's Law of Universal Gravitation $F = G \frac{m_1 m_2}{r^2}$ will be discussed in Chapter I. It relates the gravitational force F between point masses m_1 and m_2 separated by a distance r . G is Newton's Universal constant. What are the SI units of G ?

■ Problem A.6

Suppose volume V varies with time t by the relation: $V = \alpha t^2 + \beta t + \gamma$. What are the SI units of the constants α , β and γ ?

■ Problem A.7

One acre is $43\,560 \text{ ft}^2$. How many acres are in a square mile? How many square meters in an acre?

■ Problem A.8

Some units are obscure. In horse racing the length unit of furlongs is used. There are 8 furlongs to a mile. A fortnight is an obscure unit for time, where 1 fortnight = 14 days.

Suppose these units are used to study velocity and acceleration. Convert $1 \frac{\text{furlong}}{\text{fortnight}}$ and $1 \frac{\text{furlong}}{\text{fortnight}^2}$ to SI units.