

Physics 2426 - Formula List

■ Coulomb's Law

$$F = k_e \frac{|Q_1| |Q_2|}{r^2} \quad (\text{magnitude of the force})$$

$$\vec{F}_{12} = k_e Q_1 Q_2 \frac{\hat{r}_{12}}{r_{12}^2} \quad \text{where} \quad \frac{\hat{r}_{12}}{r_{12}^2} = \frac{\vec{r}_{12}}{r_{12}^3}$$

and where \vec{r}_{12} is the vector from Q_1 to Q_2 .

Charge quantization: $Q = n e$, n is an integer.

■ Electric Field

$$\vec{E} = \vec{F} / q_0 \quad \text{or} \quad \vec{E} = \lim_{q_0 \rightarrow 0} \vec{F} / q_0$$

$$\vec{F} = Q \vec{E} \quad (\text{force on charge } Q)$$

Point Charge: $\vec{E} = k_e Q \frac{\hat{r}}{r^2} = k_e Q \frac{\vec{r}}{r^3}$, $E = k_e \frac{|Q|}{r^2}$

Discrete: $\vec{E} = k_e \sum Q_i \frac{\hat{r}_i}{r_i^2} = k_e \sum Q_i \frac{\vec{r}_i}{r_i^3}$

Continuous: $\vec{E} = k_e \int \frac{\hat{r}}{r^2} dq = k_e \int \frac{\vec{r}}{r^3} dq$

$$\vec{E} = k_e Q \frac{z_0}{(R^2 + z_0^2)^{3/2}} \hat{z} \quad (\text{Field of uniform}$$

ring in the xy -plane with the center at origin)

■ Electric Flux

$$\Phi = \int \vec{E} \cdot d\vec{A}$$

For uniform field and flat surface: $\Phi = \vec{E} \cdot \vec{A}$

Dot Product:

$$\vec{A} \cdot \vec{B} = A B \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

■ **Gauss's Law** $\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} Q_{\text{in}}$

Spherical Symmetry:

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} Q_{\text{in}} \implies E 4\pi r^2 = \frac{1}{\epsilon_0} Q_{\text{in}}$$

$$\implies \vec{E} = k_e \frac{Q_{\text{in}}}{r^2} \hat{r}.$$

Cylindrical Symmetry:

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} Q_{\text{in}} \implies E 2\pi r \ell = \frac{1}{\epsilon_0} Q_{\text{in}}$$

$$\implies \vec{E} = \frac{1}{2\pi \epsilon_0} \frac{Q_{\text{in}}/\ell}{r} \hat{r}.$$

■ **Voltage and Potential Energy**

$$V = U/q_0, \Delta U = Q\Delta V$$

For a charge distribution: $U = k_e \sum_{i<j} \frac{Q_i Q_j}{r_{ij}}$

Conservation of Energy:

$$0 = \Delta K + \Delta U \quad \text{where } K = \frac{1}{2} m v^2$$

■ **Voltage due to Charges**

Point Charge: $V = k_e \frac{Q}{r}$

Discrete: $V = k_e \sum \frac{Q_i}{r_i}$

Continuous: $V = k_e \int \frac{dq}{r}$

■ **Voltage and Electric Field**

$$\Delta V = -\int \vec{E} \cdot d\vec{r} \quad \Delta V = -\vec{E} \cdot \Delta \vec{r} \quad (\text{uniform } \vec{E})$$

$$E_x = -\frac{\partial V}{\partial x} \quad \text{and for y and z. also } E_r = -\frac{\partial V}{\partial r}$$

■ **Conductors in Electrostatics**

Inside: $\vec{E} = \vec{0}$, voltage is const., no excess charge.

At surface: $\vec{E} \perp \text{surface}$ and $E = \sigma / \epsilon_0$

■ **Capacitance** $Q = C V$, C is the capacitance.

$\kappa \geq 1$ is the dielectric constant.

$C = \kappa C_0$, where C_0 is the empty capacitance.

Parallel plate: $C_0 = \frac{\epsilon_0 A}{d}$

Spherical: $C_0 = \frac{1}{k_e \left(\frac{1}{a} - \frac{1}{b} \right)}$

Cylindrical: $C_0 = \frac{2\pi\epsilon_0\ell}{\ln(b/a)}$

■ **Energy**

$U = \frac{1}{2} C V^2 = \frac{Q^2}{2C} = \frac{1}{2} Q V$ (energy in a cap.)

$u = \frac{\epsilon_0}{2} E^2 = \frac{\text{Energy}}{\text{Volume}}$ (energy density in a field)

■ **Electric Dipoles**

Dipole Moment: $\vec{p} = Q \vec{d}$, \vec{d} is from $-Q$ to Q

Torque: $\vec{\tau} = \vec{p} \times \vec{E}$, $\tau = p E \sin \theta$

Potential Energy: $U = -\vec{p} \cdot \vec{E}$

■ **Current and Current Density**

$I = \frac{dQ}{dt}$ is the current through surface.

\vec{J} is the current density. $I = \int_{\text{surface}} \vec{J} \cdot d\vec{A}$

$I = J A$ (when A is cross-section of a wire)

■ **Drift Velocity** $I = n |q| v_d A$, $\vec{J} = n q \vec{v}_d$

n is # of charge carriers per vol., q is charge of charge carriers and \vec{v}_d is drift velocity.

■ **Ohm's Law** $V = IR$, $R = \frac{\rho L}{A}$, $\vec{J} = \sigma \vec{E}$

$\sigma = \text{conductivity}$, $\rho = \frac{1}{\sigma} = \text{resistivity}$

■ **Temperature Dependence**

$\Delta R = \alpha R_0 \Delta T$ or $R = R_0 [1 + \alpha (T - T_0)]$,

$\Delta \rho = \alpha \rho_0 \Delta T$ or $\rho = \rho_0 [1 + \alpha (T - T_0)]$

■ **Power** $\mathcal{P} = VI$

For a resistor: $\mathcal{P} = VI = I^2 R = \frac{V^2}{R}$

■ **Combinations of Resistors**

Series: $R_{\text{eq}} = R_1 + R_2 + \dots$

Parallel: $R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots \right)^{-1}$

Node Reduction: $R'_{ij} = \frac{R_i R_j}{R_{\parallel}}$

where $R_{\parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$

■ **Combinations of Capacitors**

Series: $C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots \right)^{-1}$

Parallel: $C_{\text{eq}} = C_1 + C_2 + \dots$

■ **Kirchhoff's Rules**

Junction Rule: $\sum I_{\text{in}} = \sum I_{\text{out}}$

Loop Rule: $0 = \sum \Delta V$

■ **Cross or Vector Product**

$\vec{A} \times \vec{B} = \hat{x} (A_y B_z - A_z B_y)$

$+ \hat{y} (A_z B_x - A_x B_z) + \hat{z} (A_x B_y - A_y B_x)$

$\vec{A} \times \vec{B} = AB \sin \theta \hat{u}$, get \hat{u} by right hand rule

■ **Magnetic Force on Particle** $\vec{F} = Q \vec{v} \times \vec{B}$

$\vec{v} \perp$ to uniform $\vec{B} \implies$ circle with $r = \frac{mv}{|Q|B}$

■ **Magnetic Force on Wire**

$\vec{F} = I \int d\vec{r} \times \vec{B}$ (force on wire)

$\vec{F} = I \vec{\ell} \times \vec{B}$ (straight segment, uniform field)

$V_{\text{Hall}} = v_d B L$ (Hall Voltage)

■ **Magnetic Dipoles**

Dipole Moment: $\vec{\mu} = N I \vec{A}$ (N is # of turns)

Torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$, $\tau = \mu B \sin \theta$

Potential Energy: $U = -\vec{\mu} \cdot \vec{B}$

■ **Biot-Savart Law** $\vec{B} = \frac{\mu_0 I}{4\pi} \int d\vec{s} \times \frac{\hat{r}}{r^2}$

$\vec{B} = \hat{z} \frac{\mu_0 I}{4\pi R} \theta$ (at center of arc in xy -plane)

$\vec{B} = \hat{z} \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$ (z from center of circle)

$B = \frac{\mu_0 I}{4\pi a} (\sin \phi_2 - \sin \phi_1)$
 $= \frac{\mu_0 I}{4\pi a} \left(\frac{x_2}{\sqrt{x_2^2 + a^2}} - \frac{x_1}{\sqrt{x_1^2 + a^2}} \right)$ (for segment)

$B = \frac{\mu_0 I}{2\pi r}$ (distance r from long wire)

■ **Ampere's Law** $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{inside}}$

$B = \mu_0 n I$, $n = \frac{\# \text{ of turns}}{\text{length}}$ (inside long solenoid)

■ **Magnetic Flux** $\Phi_m = \int \vec{B} \cdot d\vec{A}$

For uniform field and flat surface: $\Phi = \vec{B} \cdot \vec{A}$

Faraday's Law $\mathcal{E} = -N \frac{d\Phi}{dt}$, $\overline{\mathcal{E}} = -N \frac{\Delta\Phi}{\Delta t}$

AC Gen.: $\Phi = B A \cos \omega t$

$\implies \mathcal{E} = N B A \omega \sin \omega t$

■ **Motional EMF**

moving rod: $\mathcal{E} = B \ell v$ ($\vec{B} \perp \vec{v} \perp \text{rod}$)

rotating rod: $\mathcal{E} = \frac{1}{2} B \ell^2 \omega$ ($\vec{B} \parallel \text{axis} \perp \text{rod}$)

■ **Lenz's Law**

Dir. of Φ is dir. of field through loop.

$\frac{d\Phi}{dt}$ is same as (opposite to) dir. of Φ

when increasing (decreasing).

Φ_{induced} is opposite to $\frac{d\Phi}{dt}$.

Get direction of \mathcal{E} by Right Hand Rule.

■ **Maxwell's Equations**

$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} Q_{\text{in}}$

$\oint \vec{B} \cdot d\vec{A} = 0$

$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{in}} + \mu_0 \epsilon_0 \frac{d\Phi_e}{dt}$

$\oint \vec{E} \cdot d\vec{r} = - \frac{d\Phi_m}{dt}$

■ **Inductance**

Mutual Inductance: $\mathcal{E}_2 = -M \frac{dI_1}{dt}$

(Self) Inductance: $\mathcal{E} = -L \frac{dI}{dt}$

Long Solenoid: $L = \mu_0 n^2 A \ell = \mu_0 \frac{N^2}{\ell} A$

■ **Energy in Inductor** $U = \frac{1}{2} L I^2$

■ **Energy Density**

$$u = \frac{1}{2\mu_0} B^2 \text{ (in magnetic field)}$$

$$u = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 \text{ (in electromagnetic field)}$$

■ **RC Circuits** $\tau = RC = \text{time const.}, \mathcal{E} = \text{EMF}$

Discharging: $Q(t) = Q_0 e^{-t/\tau}$

Charging: $Q(t) = C\mathcal{E}(1 - e^{-t/\tau})$

■ **RL Circuits** $\tau = \frac{L}{R} = \text{time const.}$

$$I(t) = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau}) \text{ (current growth)}$$

$$I(t) = I_0 e^{-t/\tau} \text{ (current decay)}$$

■ **LC Circuits** $\omega = \frac{1}{\sqrt{LC}}$

$$Q(t) = Q_{\max} \cos(\omega t + \phi)$$

■ **LCR Circuits** $\gamma = \frac{R}{2L}, \omega = \sqrt{\frac{1}{LC} - \gamma^2}$

$$Q(t) = Q_0 e^{-\gamma t} \cos(\omega t + \phi)$$

■ **General AC Circuits** $\omega = 2\pi f$

$$V_{\text{rms}} = \frac{1}{\sqrt{2}} V_{\max}, \quad I_{\text{rms}} = \frac{1}{\sqrt{2}} I_{\max}$$

$$I(t) = I_{\max} \cos \omega t, \quad V(t) = V_{\max} \cos(\omega t + \phi)$$

$$Z = \frac{V_{\max}}{I_{\max}} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \text{ (Impedance)}$$

	Z	ϕ
Just R :	R	0
Just C :	$X_C = \frac{1}{\omega C}$	$-90^\circ = -\frac{\pi}{2}$
Just L :	$X_L = \omega L$	$90^\circ = \frac{\pi}{2}$

$$\overline{\mathcal{P}} = V_{\text{rms}} I_{\text{rms}} \cos \phi \quad (\text{Average Power})$$

■ **Series RCL** $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\tan \phi = \frac{X_L - X_C}{R}, \quad \overline{\mathcal{P}} = I_{\text{rms}}^2 R$$

Resonance: $Z = Z_{\text{min}} = R \iff X_L = X_C$

$$\iff \omega = \frac{1}{\sqrt{LC}}$$

■ **Transformer** $\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$

■ **Electromagnetic Radiation in Vacuum**

$$\vec{E} = \hat{y} E_{\text{max}} \cos(kx - \omega t)$$

$$\vec{B} = \hat{z} B_{\text{max}} \cos(kx - \omega t)$$

$$\omega = 2\pi f = \frac{2\pi}{T}, \quad k = \frac{2\pi}{\lambda}, \quad f\lambda = \frac{\omega}{k} = c$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}, \quad c = \frac{E_{\text{max}}}{B_{\text{max}}} = \frac{E}{B}$$

Poynting Vector: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

Intensity = $I = \frac{\text{Power}}{\text{Area}} = \frac{U}{A \Delta t}$, $U = \text{Energy}$

$$\vec{S} = I = c \bar{u} = \frac{E_{\text{max}}^2}{2\mu_0 c}, \quad \bar{u} = \text{Ave. Energy Density}$$

■ Radiation Pressure and Momentum

Momentum carried by radiation: $p = \frac{U}{c}$

Pressure = $\frac{\text{Force}}{\text{Area}}$

	Momentum to Surface	Pressure on Surface
Perfect Absorber	$p = \frac{U}{c}$	$P = \frac{I}{c}$
Perfect Reflector	$p = 2 \frac{U}{c}$	$P = 2 \frac{I}{c}$
$\kappa =$ fraction refl.	$p = (1 + \kappa) \frac{U}{c}$	$P = (1 + \kappa) \frac{I}{c}$

■ In a Medium $f \lambda = v = c/n$

At interface: $n_1 \lambda_1 = n_2 \lambda_2$

Snell's Law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

Total Internal Refl.: $\theta_1 > \theta_c$ where $\sin \theta_c = \frac{n_2}{n_1}$

■ Geometric Optics

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}, \quad m = \frac{h'}{h} = -\frac{s'}{s}$$

Spherical Mirrors: $f = \frac{R}{2}$,

concave: $R > 0$, convex: $R < 0$, flat: $R \rightarrow \infty$

Thin Lenses: $f > 0$ (converging), $f < 0$ (diverging)

Spher. Int.: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$, $m = \frac{h'}{h} = -\frac{n_1}{n_2} \frac{s'}{s}$

Flat Interface: $\frac{s'}{s} = -\frac{n_2}{n_1}$, $m = \frac{h'}{h} = 1$

Lensmaker Formula: $\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

■ Interference and Diffraction

$\tan \theta = \frac{y}{L}$, Small θ or when $y \ll L \implies \sin \theta = \frac{y}{L}$

Double Slit: Intensity: $I = I_{\max} \cos^2\left(\frac{\pi d \sin \theta}{\lambda}\right)$

Const. Int.: $d \sin \theta = m \lambda$

Dest. Int.: $d \sin \theta = \left(m + \frac{1}{2}\right) \lambda$

Diffraction Grating:

Const. Int: $d \sin \theta = m \lambda$, Dest. Int. elsewhere

Single Slit: Destr. Int.: $a \sin \theta = m \lambda$, $m \neq 0$

Thin Films	Constructive Interference	Destructive Interference
$n < n'$	$2 t = m \frac{\lambda}{n}$	$2 t = \left(m + \frac{1}{2}\right) \frac{\lambda}{n}$
$n > n'$	$2 t = \left(m + \frac{1}{2}\right) \frac{\lambda}{n}$	$2 t = m \frac{\lambda}{n}$

Phase Shift on Reflection:

$n < n' \implies 180^\circ$ shift, $n > n' \implies$ no shift

■ **Polarization**

$I = \frac{1}{2} I_0$, $I = I_0 \cos^2 \theta$, $\tan \theta_P = \frac{n_2}{n_1}$ (Pol. \perp)