

# Physics 2425 - Formula List

## 1D Kinematics

General 1D Motion:  $x$  as a function of  $t$

$$\bar{v} = \frac{\Delta x}{\Delta t}, \quad v = \frac{dx}{dt}, \quad \bar{a} = \frac{\Delta v}{\Delta t}, \quad a = \frac{dv}{dt}$$

Constant Vel.:  $x(t) = x_0 + vt \implies \Delta x = vt$

Constant Acc.:  $v(t) = v_0 + at$  and  $x(t) = x_0 + v_0 t + \frac{1}{2} at^2$

$$v = v_0 + at \quad \Delta x = \frac{1}{2}(v_0 + v)t$$

$$\Delta x = v_0 t + \frac{1}{2} at^2 \quad v^2 - v_0^2 = 2a \Delta x$$

Free Fall:  $x \rightarrow y$  ( $y$  is up) and  $a = -g$

**General Vectors**  $\vec{A} = A_x \hat{x} + A_y \hat{y} = \langle A_x, A_y \rangle$

**Mag. & Dir. angle**  $\implies$  Components  $A_x = A \cos \theta$  and  $A_y = A \sin \theta$ .

**Components**  $\implies$  **Mag. & Dir. angle**

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and } \theta = \begin{cases} \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{for } A_x > 0 \\ 180^\circ + \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{for } A_x < 0 \end{cases}$$

**General 2D Kinematics**  $\vec{r}$  as a function of  $t$

$$\bar{v} = \frac{\Delta \vec{r}}{\Delta t}, \quad \vec{v} = \frac{d\vec{r}}{dt}, \quad \bar{a} = \frac{\Delta \vec{v}}{\Delta t}, \quad \vec{a} = \frac{d\vec{v}}{dt}, \quad \text{Ave. Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Constant Acc.:  $\vec{v}(t) = \vec{v}_0 + \vec{a}t$  and  $\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a}t^2$

## Projectiles

Horizontal:  $a_x = 0 \implies v_x$  is const. Vertical:  $a_y = -g$

$v_{0x} = v_0 \cos \theta$  and  $v_{0y} = v_0 \sin \theta$ .

$$R = \frac{v_0^2}{g} \sin(2\theta), \quad (R = \Delta x \text{ when } \Delta y = 0)$$

**Relative Motion**  $\vec{v} = \vec{v}' + \vec{v}_0$

## Newton's Laws

First Law:  $\vec{v}$  is const., unless net force.

Second Law:  $\vec{F}_{\text{net}} = m\vec{a}$

Third Law:  $\vec{F}_{12} = -\vec{F}_{21}$

Weight  $\propto$  Mass:  $W = mg$

**Friction between surfaces**  $f_s \leq \mu_s N$  (static),  $f_k = \mu_k N$  (kinetic)

## Circular Motion

Uniform Circular Motion:  $a_c = \frac{v^2}{r}$ , Also  $v = \frac{2\pi r}{T} \implies a_c = \left(\frac{2\pi}{T}\right)^2 r$

General Circular Motion:  $a_c = \frac{v^2}{r}$ ,  $a_t = \frac{dv}{dt}$

**Accelerated Frames**  $\vec{g}_{\text{art}} = -\vec{a}$  (artificial gravity)

accelerated frame  $\implies$  false force opposite acc.

## Dot or Scalar Product

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad \text{where } A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

**Work**  $W = \int \vec{F} \cdot d\vec{r}$

const. force:  $W = F \Delta x$  (1D)  $W = \vec{F} \cdot \Delta \vec{r}$  (2D or 3D)

$W_{\text{grav}} = -mg \Delta y$  (work done by gravity)

$W = \int_{x_i}^{x_f} F(x) dx$  (variable force in 1D)

## Springs: Hooke's Law and Work

$F = -kx$  (Hooke's Law),  $W = -\frac{1}{2}k(x_f^2 - x_i^2)$

## Work-Energy Theorem

$W_{\text{net}} = \Delta K$   
 $W_{\text{net}}$  (net work).  $K = \frac{1}{2}mv^2$  (kinetic energy)

$\vec{F}$  is conservative  $\iff 0 = \oint \vec{F} \cdot d\vec{r}$

conservative forces  $\implies \Delta U = -W$  ( $U$  is potential energy)

Gravity:  $U = mgy$  Spring:  $U = \frac{1}{2}kx^2$

$W_{\text{nc}}$  is work of all nonconservative forces.

$E = E^{\text{mech}} = K_{\text{tot}} + U_{\text{tot}} \implies E_i + W_{\text{nc}} = E_f$ ,  $W_{\text{nc}} = 0 \implies E_i = E_f$

$W_{\text{nc}} = 0$ , one mass, gravity is only cons. force

$$\implies E_{\text{bottom}} = E_{\text{top}} \implies v_{\text{bottom}}^2 = v_{\text{top}}^2 + 2gh$$

Power:  $\mathcal{P} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$

## Potential Energy $\implies$ Force

1D:  $F = -\frac{d}{dx}U$ , 3D:  $F_x = -\frac{\partial}{\partial x}U$ ,  $F_y = -\frac{\partial}{\partial y}U$ ,  $F_z = -\frac{\partial}{\partial z}U$

## Momentum and Impulse-Momentum Theorem

$\vec{p} = m\vec{v}$  (mom.)  $\vec{I} = \int_{t_i}^{t_f} \vec{F} dt$  (impulse)

$$\vec{F}_{\text{net}} \Delta t = \vec{I}_{\text{net}} = \Delta \vec{p} = m(\vec{v}_f - \vec{v}_i)$$

## Second Law for a Particle and System

particle:  $\vec{F}_{\text{net}} = m\vec{a}$   $\vec{F}_{\text{net}} = \frac{d}{dt}\vec{p}$

system:  $\vec{F}_{\text{net}}^{\text{ext}} = M\vec{a}_{\text{cm}}$   $\vec{F}_{\text{net}}^{\text{ext}} = \frac{d}{dt}\vec{p}_{\text{tot}}$

$$M = m_1 + m_2 + \dots$$

$$\vec{r}_{\text{cm}} = \frac{1}{M}(m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots)$$

$$x_{\text{cm}} = \frac{1}{M}(m_1 x_1 + m_2 x_2 + \dots) \quad (\text{also } y \text{ and } z)$$

$$\vec{p}_{\text{tot}} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots = M\vec{v}_{\text{cm}}$$

## Conservation of Momentum

$$\vec{F}_{\text{net}}^{\text{ext}} = 0 \implies \Delta \vec{p}_{\text{tot}} = 0 \implies \vec{p}_{\text{tot},i} = \vec{p}_{\text{tot},f}$$

$$F_{\text{net},x}^{\text{ext}} = 0 \implies \Delta p_{\text{tot},x} = 0 \implies p_{\text{tot},i,x} = p_{\text{tot},f,x}$$

## Collisions

$$\vec{p}_{\text{tot},i} = \vec{p}_{\text{tot},f} \implies m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} = m_1 \vec{v}_{1,f} + m_2 \vec{v}_{2,f}$$

Elastic  $\iff K_{\text{tot},i} = K_{\text{tot},f}$

1D Elastic trick:  $K_{\text{tot},i} = K_{\text{tot},f} \implies v_{1,i} + v_{1,f} = v_{2,i} + v_{2,f}$

Totally Inelastic:  $\vec{v}_{1,f} = \vec{v}_{2,f} = \vec{v}_f \implies m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} = (m_1 + m_2) \vec{v}_f$

## General Rotations about fixed axis:

$$\bar{\omega} = \frac{\Delta \theta}{\Delta t}, \quad \omega = \frac{d\theta}{dt}, \quad \bar{\alpha} = \frac{\Delta \omega}{\Delta t}, \quad \alpha = \frac{d\omega}{dt}$$

## Constant Angular Accelation

$$\omega = \omega_0 + \alpha t \quad \Delta \theta = \frac{1}{2}(\omega + \omega_0)t$$

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad \omega^2 - \omega_0^2 = 2 \alpha \Delta \theta$$

## Rotational and Linear Quantities

$\vec{v} = r\omega \hat{u}_t$  or  $v_t = r\omega$ ,  $v_c = 0$

$\vec{a} = r\alpha \hat{u}_t + \omega^2 r \hat{u}_c$  or  $a_t = r\alpha$ ,  $a_c = \omega^2 r$

$$\alpha = 0 \iff \omega = \text{const} = \frac{2\pi}{T}$$

### ■ Moment of Inertia

$I$  for a distribution -  $r$  is  $\perp$  dist. from axis

$$\text{Discrete: } I = \sum m_i r_i^2, \quad \text{Continuous: } I = \int r^2 dm$$

**Perpendicular-axis Theorem:** Planar object in  $xy$ -plane:  $I_z = I_x + I_y$

**Parallel-axis Theorem:**  $I = I_{\text{cm}} + M D^2$

**Moments for uniform bodies:**

Thin rod about  $\perp$  axis

$$\text{thru. end: } I = \frac{1}{3} M L^2, \quad \text{thru. center: } I = \frac{1}{12} M L^2$$

$$a \times b \text{ rectangular plate about } \perp \text{ axis thru. center: } I = \frac{1}{12} M (a^2 + b^2)$$

Sphere about axis thru. center:

$$\text{thin shelled hollow: } I = \frac{2}{3} M R^2, \quad \text{solid: } I = \frac{2}{5} M R^2$$

Hoop about  $\perp$  axis thru. center:  $I = M R^2$

(same as thin-shelled hollow cylinder)

Disk about  $\perp$  axis thru. center:  $I = \frac{1}{2} M R^2$  (same as solid cylinder)

### ■ Rotational Energy

$$K = \frac{1}{2} I \omega^2, \quad U = M g y_{\text{cm}}, \quad K_{\text{tot}} = \frac{1}{2} I_{\text{cm}} \omega^2 + \frac{1}{2} M v_{\text{cm}}^2$$

**■ Cross or Vector Product**  $\vec{A} \times \vec{B} = \hat{u} AB \sin \theta$ , right hand rule  $\Rightarrow \hat{u}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{y} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{z} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

### ■ Torque

About Origin:  $\vec{\tau} = \vec{r} \times \vec{F}$ , About Axis:  $\tau = r F_{\perp} = r_{\perp} F = r F \sin \theta$

Torque due to gravity:  $\vec{\tau}_{\text{gravity}} = \vec{r}_{\text{cm}} \times M \vec{g}$

### ■ Angular Momentum of Particle

About Origin:  $\vec{L} = \vec{r} \times \vec{p}$ , About Axis:  $L = r p_{\perp} = r_{\perp} p = r p \sin \theta$

### ■ General Rigid Body Dynamics

2nd Law:  $\tau_{\text{net}} = I \alpha$  and  $\tau_{\text{net}} = \frac{dL}{dt}$

Angular Momentum:  $L = I \omega$

**■ System of Particles**  $\vec{\tau}_{\text{net}}^{\text{ext}} = \frac{d\vec{L}_{\text{tot}}}{dt}$

$\vec{\tau}_{\text{net}}^{\text{ext}} = 0 \implies \Delta \vec{L}_{\text{tot}} = 0$  (Conservation)

**■ Equilibrium**  $\vec{F}_{\text{net}} = \vec{0}$  and  $\vec{\tau}_{\text{net}} = \vec{0}$

### ■ Newton's Law of Gravity

Magnitude:  $F = G \frac{m_1 m_2}{r^2}$ , Vector:  $\vec{F}_{21} = -G \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$

Discrete Distribution:  $\vec{F} = -G m \sum \frac{m_i}{r_i^2} \hat{r}_i$

Continuous Distribution:  $\vec{F} = -G m \int \frac{\hat{r}}{r^2} dm$

Sph. Shell:  $F = G \frac{M m}{r^2}$  ( $r > R$ ),  $F = 0$  ( $r < R$ )

$g = G \frac{M}{R^2}$  (at surface of spherical planet)

### ■ Gravitational Potential Energy

Two masses:  $U = -G \frac{M m}{r}$ , Several masses:  $U = -G \sum_{i < j} \frac{m_i m_j}{r_{ij}}$

Escape speed:  $v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$

**■ Circular Orbits**  $v^2 = G \frac{M}{r}$  and  $T^2 = \frac{4\pi^2}{GM} r^3$

**■ Simple Harmonic Motion**  $\frac{d^2x}{dt^2} = -\omega^2 x$ ,  $\omega = 2\pi f = \frac{2\pi}{T}$

$$x(t) = A \cos(\omega t + \phi) = x_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t$$

**■ Energy**  $E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \begin{cases} \frac{1}{2} k A^2 \\ \frac{1}{2} m v_{\text{max}}^2 \end{cases}$  (mass/spring)

$v = \pm \omega \sqrt{A^2 - x^2}$  and  $v_{\text{max}} = \omega A$  (in general)

### ■ Examples of Simple Harmonic Motion

Mass/Spring:  $\omega = \sqrt{k/m}$

Physical Pendulum:  $\omega = \sqrt{m g d / I}$

Simple Pendulum:  $\omega = \sqrt{g/L}$

**■ 1D Wave Equation**  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2}$

(General Solution)  $u(x, t) = f(x - vt) + g(x + vt)$

**■ Sinusoidal Waves**  $u(x, t) = A \cos(kx \pm \omega t + \phi)$

$$\lambda = \frac{2\pi}{k}, \quad f = \frac{\omega}{2\pi}, \quad v = f \lambda = \frac{\omega}{k}$$

**■ Waves on a String**  $u(x, t) \Rightarrow y(x, t)$

Speed:  $v = \sqrt{T/\mu}$  where  $T$  = Tension, Power:  $\mathcal{P} = \frac{1}{2} \mu A^2 \omega^2 v$

### ■ Temperature and Thermal Expansion

$T_F = \frac{9}{5} T_C + 32$ ,  $\Delta T_F = \frac{9}{5} \Delta T_C$ ,  $T_K = T_C + 273$

$\Delta L = \alpha L_0 \Delta T$ ,  $\Delta V = \beta V_0 \Delta T$ ,  $\beta = 3\alpha$  (for solid)

**■ Heat**  $Q$  = Heat added to system

$Q = m c \Delta T$  (Temp. change),  $Q = \pm m L$  (phase change)

**■ Ideal Gas Law**  $P V = N k T$  and  $P V = n R T$

$n$  = # of moles,  $N = N_A n$  = # of molecules

Masses:  $m_{\text{tot}} = n m_{\text{mole}} = N m_{\text{molecule}}$

**■ Work**  $W = \int P dV = \pm \text{Area}$  (done by system)

Constant  $P$ :  $W = P \Delta V$ ,

Ideal gas at constant  $T$ :  $W = n R T \ln(V_f / V_i)$

### ■ First Law

$\Delta U = Q - W$ ,  $dU = dQ - dW$ , ( $d$  is inexact differential)

**■ Entropy**  $dS = \frac{dQ}{T} \implies \Delta S = \int \frac{dQ}{T}$

Const.  $T$ :  $\Delta S = \frac{Q}{T}$ , Changing  $T$ :  $\Delta S = m c \ln \frac{T_f}{T_i}$

**■ Second Law** For a thermally isolated system:  $\Delta S_{\text{tot}} \geq 0$

**■ Heat Engines**  $Q_H = Q_C + W$ , Efficiency:  $e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$

Max. Eff.:  $e_{\text{max}} = 1 - \frac{T_C}{T_H}$  (Carnot Engine is H.E. of max. eff.)