

Physics 2425 - Formula List

■ 1D Kinematics

General 1D Motion: x as a function of t

$$\bar{v} = \frac{\Delta x}{\Delta t}, \quad v = \frac{dx}{dt}, \quad \bar{a} = \frac{\Delta v}{\Delta t}, \quad a = \frac{dv}{dt}$$

Constant Vel.: $x(t) = x_0 + vt \implies \Delta x = vt$

Constant Acceleration:

$$v(t) = v_0 + at \quad \text{and} \quad x(t) = x_0 + v_0 t + \frac{1}{2} at^2$$

$$v = v_0 + at \quad \Delta x = \frac{1}{2} (v_0 + v) t$$

$$\Delta x = v_0 t + \frac{1}{2} at^2 \quad v^2 - v_0^2 = 2a \Delta x$$

Free Fall: $x \rightarrow y$ (y is up) and $a = -g$

■ General Vectors

$$\vec{A} = A_x \hat{x} + A_y \hat{y} = \langle A_x, A_y \rangle$$

Mag. & Dir. angle \implies Components

$$A_x = A \cos \theta \quad \text{and} \quad A_y = A \sin \theta.$$

Components \implies Mag. & Dir. angle

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \begin{cases} \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{when } A_x > 0 \\ 180^\circ + \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{when } A_x < 0 \end{cases}$$

■ 2D Kinematics

General 2D Motion:

\vec{r} as a function of t

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}, \quad \vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}, \quad \vec{a} = \frac{d\vec{v}}{dt}$$

$$\text{Ave. Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Constant Acceleration:

$$\vec{v}(t) = \vec{v}_0 + \vec{a} t$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

■ Projectiles

Horizontal Motion:

$$a_x = 0 \implies v_x \text{ is const.}$$

Vertical Motion: $a_y = -g$

$$v_{0x} = v_0 \cos \theta \quad \text{and} \quad v_{0y} = v_0 \sin \theta.$$

$$R = \frac{v_0^2}{g} \sin(2\theta), \quad (R = \Delta x \text{ when } \Delta y = 0)$$

Relative Motion: $\vec{v} = \vec{v}' + \vec{v}_0$

■ Newton's Laws

First Law: \vec{v} is const., unless net force.

Second Law: $\vec{F}_{\text{net}} = m \vec{a}$

Third Law: $\vec{F}_{12} = -\vec{F}_{21}$

Weight \propto Mass: $W = m g$

■ Friction between surfaces

$f_s \leq \mu_s N$ (static) $f_k = \mu_k N$ (kinetic)

■ Circular Motion

Uniform Circular Motion: $a_c = \frac{v^2}{r}$

$$\text{Also } v = \frac{2\pi r}{T} \implies a_c = \left(\frac{2\pi}{T}\right)^2 r$$

Gen. Circular Motion: $a_c = \frac{v^2}{r}$, $a_t = \frac{dv}{dt}$

■ Accelerated Frames

accelerated frame \implies false force opposite acc.

$\vec{g}_{\text{art}} = -\vec{a}$ (artificial gravity)

■ Dot or Scalar Product

$\vec{A} \cdot \vec{B} = AB \cos \theta$ where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

■ **Work** $W = \int \vec{F} \cdot d\vec{r}$

$W = F \Delta x$ (const. force in 1D)

$W = \vec{F} \cdot \Delta \vec{r}$ (const. force in 2D or 3D)

$W_{\text{grav}} = -m g \Delta y$ (work done by gravity)

$W = \int_{x_i}^{x_f} F(x) dx$ (variable force in 1D)

■ **Work-Energy Theorem** $W_{\text{net}} = \Delta K_{\text{tot}}$

W_{net} (net work). $K = \frac{1}{2} m v^2$ (kinetic energy)

\vec{F} is cons. $\iff W = \int \vec{F} \cdot d\vec{r}$ is indep. of path.

cons. forces $\implies \Delta U = -W$ (U is pot. energy)

W_{nc} is work of all noncons. forces.

$E^{\text{mech}} = K_{\text{tot}} + U_{\text{tot}}$ (total mechanical energy)

$W_{\text{nc}} = \Delta E^{\text{mech}} = \Delta K_{\text{tot}} + \Delta U_{\text{tot}}$

$W_{\text{nc}} = 0 \implies E_i = E_f$, where $E = E_{\text{mech}}$

$U = m g y$ for gravity

$W_{\text{nc}} = 0$, one mass, gravity is only cons. force

$\implies E_{\text{bottom}} = E_{\text{top}} \implies v_{\text{bottom}}^2 = v_{\text{top}}^2 + 2 g h$

Power: $\mathcal{P} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$

Springs and Hooke's Law

$$F = -k x \text{ (Hooke's Law)}$$

$$W = -\frac{1}{2} k (x_f^2 - x_i^2) \text{ and } U = \frac{1}{2} k x^2$$

■ Potential Energy \Rightarrow Force

$$1D: F = -\frac{d}{dx} U$$

$$3D: F_x = -\frac{\partial}{\partial x} U, F_y = -\frac{\partial}{\partial y} U, F_z = -\frac{\partial}{\partial z} U$$

■ Momentum and Impulse-Momentum Theorem

$$\vec{p} = m \vec{v} \text{ (mom.)} \quad \vec{I} = \int_{t_i}^{t_f} \vec{F} dt \text{ (impulse)}$$

$$\vec{F}_{\text{net}} \Delta t = \vec{I}_{\text{net}} = \Delta \vec{p} = m(\vec{v}_f - \vec{v}_i)$$

■ Second Law for a Particle and System

$$\text{particle: } \vec{F}_{\text{net}} = m \vec{a} \quad \vec{F}_{\text{net}} = \frac{d}{dt} \vec{p}$$

$$\text{system: } \vec{F}_{\text{net}}^{\text{ext}} = M \vec{a}_{\text{cm}} \quad \vec{F}_{\text{net}}^{\text{ext}} = \frac{d}{dt} \vec{p}_{\text{tot}}$$

$$M = m_1 + m_2 + \dots$$

$$\vec{r}_{\text{cm}} = \frac{1}{M} (m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots)$$

$$x_{\text{cm}} = \frac{1}{M} (m_1 x_1 + m_2 x_2 + \dots) \text{ (also } y \text{ and } z)$$

$$\vec{p}_{\text{tot}} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots = M \vec{v}_{\text{cm}}$$

■ Conservation of Momentum

$$\vec{F}_{\text{net}}^{\text{ext}} = 0 \Rightarrow \Delta \vec{p}_{\text{tot}} = 0 \Rightarrow \vec{p}_{\text{tot},i} = \vec{p}_{\text{tot},f}$$

$$F_{\text{net},x}^{\text{ext}} = 0 \Rightarrow \Delta p_{\text{tot},x} = 0$$

$$\Rightarrow p_{\text{tot},i,x} = p_{\text{tot},f,x}$$

■ Collisions

$$\vec{p}_{\text{tot},i} = \vec{p}_{\text{tot},f} \Rightarrow$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$\text{Totally Inelastic} \iff \vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_{1f}$$

$$\Rightarrow m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$$

$$\text{Elastic} \iff K_{\text{tot},i} = K_{\text{tot},f}$$

Trick for 1D Elastic:

$$K_{\text{tot},i} = K_{\text{tot},f} \Rightarrow v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

■ **General Rotations about fixed axis:**

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \quad \omega = \frac{d\theta}{dt} \quad \bar{\alpha} = \frac{\Delta\omega}{\Delta t} \quad \alpha = \frac{d\omega}{dt}$$

■ **Constant Angular Accelation**

$$\omega = \omega_0 + \alpha t \quad \Delta\theta = \frac{1}{2} (\omega + \omega_0) t$$

$$\Delta\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad \omega^2 - \omega_0^2 = 2 \alpha \Delta\theta$$

■ **Rotational and Linear Quantities**

$$\vec{v} = r \omega \hat{u}_t \quad \text{or} \quad v_t = r \omega, \quad v_c = 0$$

$$\vec{a} = r \alpha \hat{u}_t + \omega^2 r \hat{u}_c \quad \text{or} \quad a_t = r \alpha, \quad a_c = \omega^2 r$$

$$\alpha = 0 \iff \omega = \text{const} = \frac{2\pi}{T}$$

■ **Moment of Inertia**

I for a distribution - r is \perp dist. from axis

$$\text{Discrete: } I = \sum m_i r_i^2 \quad \text{Cont.: } I = \int r^2 dm$$

Perpendicular-axis Theorem:

for a planar object in xy -plane: $I_z = I_x + I_y$

Parallel-axis Theorem: $I = I_{\text{cm}} + M D^2$

Moments for uniform bodies:

Thin rod about \perp axis

$$\text{through end: } I = \frac{1}{3} M L^2$$

$$\text{through center: } I = \frac{1}{12} M L^2$$

$a \times b$ rectangular plate about \perp axis

$$\text{through center } I = \frac{1}{12} M (a^2 + b^2)$$

Sphere about axis through center

thin shelled hollow: $I = \frac{2}{3} M R^2$

solid: $I = \frac{2}{5} M R^2$

Hoop about \perp axis through center: $I = M R^2$

(same as thin-shelled hollow cylinder)

Disk about \perp axis through center: $I = \frac{1}{2} M R^2$

(same as solid cylinder)

■ Rotational Energy

$$K = \frac{1}{2} I \omega^2, \quad U = M g y_{\text{cm}}$$

$$K_{\text{total}} = \frac{1}{2} I_{\text{cm}} \omega^2 + \frac{1}{2} M v_{\text{cm}}^2$$

■ Cross or Vector Product

$$\vec{A} \times \vec{B} = \hat{x} (A_y B_z - A_z B_y) \\ + \hat{y} (A_z B_x - A_x B_z) + \hat{z} (A_x B_y - A_y B_x)$$

$$\vec{A} \times \vec{B} = A B \sin \theta \hat{u}, \quad \text{get } \hat{u} \text{ by right hand rule}$$

■ Torque

About Origin: $\vec{\tau} = \vec{r} \times \vec{F}$

About Axis: $\tau = r F_{\perp} = r_{\perp} F = r F \sin \theta$

Torque of Gravity: $\vec{\tau}_{\text{gravity}} = \vec{r}_{\text{cm}} \times M \vec{g}$

■ Angular Momentum of Particle

About Origin: $\vec{L} = \vec{r} \times \vec{p}$

About Axis: $L = r p_{\perp} = r_{\perp} p = r p \sin \theta$

■ General Rigid Body Dynamics

2nd Law: $\tau_{\text{net}} = I \alpha$ and $\tau_{\text{net}} = \frac{dL}{dt}$

Angular Momentum: $L = I \omega$

- **System of Particles** $\vec{\tau}_{\text{net}}^{\text{ext}} = \frac{d\vec{L}_{\text{tot}}}{dt}$

$$\vec{\tau}_{\text{net}}^{\text{ext}} = 0 \implies \Delta \vec{L}_{\text{tot}} = 0 \quad (\text{Conservation})$$

- **Equilibrium** $\vec{F}_{\text{net}} = \vec{0}$ and $\vec{\tau}_{\text{net}} = \vec{0}$

- **Newton's Law of Gravity**

Magnitude: $F = G \frac{m_1 m_2}{r^2}$

Vector: $\vec{F}_{21} = -G \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$

Discrete Distribution: $\vec{F} = -G m \sum \frac{m_i}{r_i^2} \hat{r}_i$

Continuous Distribution: $\vec{F} = -G m \int \frac{\hat{r}}{r^2} dm$

Sph. Shell: $F = G \frac{M m}{r^2}$ ($r > R$), $F = 0$ ($r < R$)

$g = G \frac{M}{R^2}$ (at surface of spherical planet)

- **Gravitational Potential Energy**

Between 2 masses: $U = -G \frac{M m}{r}$

Several masses: $U = -G \sum_{i < j} \frac{m_i m_j}{r_{ij}}$

Escape speed: $v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$

- **Circular Orbits**

$$v^2 = G \frac{M}{r} \quad \text{and} \quad T^2 = \frac{4\pi^2}{GM} r^3$$

- **Simple Harmonic Motion** $\frac{d^2x}{dt^2} = -\omega^2 x$

$$x(t) = A \cos(\omega t + \phi) = x_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

■ Energy

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \begin{cases} \frac{1}{2} k A^2 \\ \frac{1}{2} m v_{\max}^2 \end{cases} \quad (\text{mass/spring})$$

$$v = \pm \omega \sqrt{A^2 - x^2} \quad \text{and} \quad v_{\max} = \omega A \quad (\text{in general})$$

■ Examples of SHM

$$\text{Mass/Spring: } \omega = \sqrt{\frac{k}{m}}$$

$$\text{Physical Pendulum: } \omega = \sqrt{\frac{m g d}{I}}$$

$$\text{Simple Pendulum: } \omega = \sqrt{\frac{g}{L}}$$

■ 1D Wave Equation

$$\frac{\partial^2}{\partial x^2} u = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} u$$

$$(\text{General Solution}) \quad u(x, t) = f(x - vt) + g(x + vt)$$

■ Harmonic Waves

$$u(x, t) = A \sin(kx \pm \omega t + \phi)$$

$$\lambda = \frac{2\pi}{k}, \quad f = \frac{\omega}{2\pi}, \quad v = f\lambda = \frac{\omega}{k}$$

■ Waves on a String

$$u(x, t) \Rightarrow y(x, t)$$

$$\text{Speed: } v = \sqrt{\frac{T}{\mu}} \quad \text{where } T = \text{Tension}$$

$$\text{Power: } \mathcal{P} = \frac{1}{2} \mu A^2 \omega^2 v$$

■ Temperature

$$T_F = \frac{9}{5} T_C + 32, \quad \Delta T_F = \frac{9}{5} \Delta T_C, \quad T_K = T_C + 273$$

$$\Delta L = \alpha L_0 \Delta T, \quad \Delta V = \beta V_0 \Delta T, \quad \beta = 3\alpha \quad (\text{for solid})$$

- **Heat** Q = Heat added to system

$$Q = m c \Delta T \quad (\text{Temp. change})$$

$$Q = \pm m L \quad (\text{phase change})$$

- **Ideal Gas Law** $P V = N k T$ and $P V = n R T$

$$n = \# \text{ of moles, } N = N_A n = \# \text{ of molecules}$$

- **Work**

$$W = \int P dV = \text{Work done by system} = \pm \text{Area}$$

$$\text{Constant } P: W = P \Delta V,$$

$$\text{Ideal gas at constant } T: W = n R T \ln \frac{V_f}{V_i}$$

- **First Law** $\Delta U = Q - W$

$$dU = \underline{d}Q - \underline{d}W \quad (\underline{d} \text{ is inexact differential})$$

- **Entropy** $dS = \frac{dQ}{T} \implies \Delta S = \int \frac{dQ}{T}$

$$\text{Const. } T: \Delta S = \frac{Q}{T} \quad \text{Changing } T: \Delta S = m c \ln \frac{T_f}{T_i}$$

- **Second Law**

$$\text{For a thermally isolated system: } \Delta S_{\text{tot}} \geq 0$$

- **Heat Engines**

$$Q_H = Q_C + W$$

$$Q_H = \text{Input Heat (from hot reservoir at } T_H)$$

$$Q_C = \text{Expelled Heat (to cold reservoir at } T_C)$$

$$W = \text{Work done by engine}$$

$$\text{Efficiency: } e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

$$\text{Maximum Efficiency: } e_{\text{max}} = 1 - \frac{T_C}{T_H}$$

Carnot Engine is H.E. of max. efficiency