Physics 2425
Formula Sheet – Final

One-dimensional kinematics

- displacement: $\Delta x \equiv x_f - x_i$
- $\text{average speed} \equiv \dfrac{\text{total distance traveled}}{\text{total time}}$
- average velocity: $v_{av} \equiv \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$
- instantaneous velocity: $v \equiv \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$
- average acceleration: $a_{av} \equiv \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i}$
- instantaneous acceleration: $a \equiv \lim_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt} = \dfrac{d^2 x}{dt^2}$
- One-dimensional motion with constant acceleration:
  (1) $v = v_0 + at$
  (2) $x = x_0 + \frac{1}{2}(v_0 + v)t$
  (3) $x = x_0 + v_0 t + \frac{1}{2}at^2$
  (4) $v^2 = v_0^2 + 2a(x - x_0)$
- Free fall (positive direction for y taken to be *upward*)
  $*\ x \to y$ and $a \to -g$ in the above 4 equations of kinematics:

  (1) $v = v_0 - gt$
  (2) $y = y_0 + \frac{1}{2}(v_0 + v)t$
  (3) $y = y_0 + v_0 t - \frac{1}{2}gt^2$
  (4) $v^2 = v_0^2 - 2g(y - y_0)$

Vectors

Vectors in 2-D

If a vector $\vec{A}$ is written in *component form* as $\vec{A} = A_x \hat{x} + A_y \hat{y} = \langle A_x , A_y \rangle$ then:

- Getting magnitude and direction of $\vec{A}$ from the components:
  $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$ (magnitude of $\vec{A}$)
  $\theta = \arctan\left(\dfrac{A_y}{A_x}\right)$ (direction of $\vec{A}$)

- Getting components from magnitude and direction:

  $A_x = |\vec{A}|\cos\theta$ (x component of $\vec{A}$)
  $A_y = |\vec{A}|\sin\theta$ (y component of $\vec{A}$)
  } θ understood to be the angle that $\vec{A}$ makes with the positive x axis

Vectors in 3-D

If $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} = \langle A_x, A_y, A_z \rangle$ then:

- $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

2-D Kinematics

- position vector: $\vec{r}(t) = x(t)\hat{x} + y(t)\hat{y} = \langle x(t), y(t) \rangle$
- average velocity: $\vec{v}_{av} \equiv \frac{\Delta\vec{r}}{\Delta t} = \left\langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t} \right\rangle$
- instantaneous velocity: $\vec{v} \equiv \frac{d\vec{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$
- average acceleration: $\vec{a}_{av} \equiv \frac{\Delta\vec{v}}{\Delta t} = \left\langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t} \right\rangle$
- instantaneous acceleration:
  - $\vec{a} \equiv \frac{d\vec{v}}{dt} = \left\langle \frac{dv_x}{dt}, \frac{dv_y}{dt} \right\rangle$
  - $\vec{a} \equiv \frac{d^2\vec{r}}{dt^2} = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$
- instantaneous speed $\equiv |\vec{v}|$ (magnitude of the instantaneous velocity)

3-D kinematics

- position vector: $\vec{r}(t) = x(t)\hat{x} + y(t)\hat{y} + z(t)\hat{z} = \langle x(t), y(t), z(t) \rangle$
- average velocity: $\vec{v}_{av} \equiv \frac{\Delta\vec{r}}{\Delta t} = \left\langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t} \right\rangle$
- instantaneous velocity: $\vec{v} \equiv \frac{d\vec{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$
- average acceleration: $\vec{a}_{av} \equiv \frac{\Delta\vec{v}}{\Delta t} = \left\langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t} \right\rangle$
- instantaneous acceleration:
  - $\vec{a} \equiv \frac{d\vec{v}}{dt} = \left\langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \right\rangle$
  - $\vec{a} \equiv \frac{d^2\vec{r}}{dt^2} = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \right\rangle$
- instantaneous speed $\equiv |\vec{v}|$ (magnitude of the instantaneous velocity)

- Projectile Motion

  - x direction (motion with constant velocity):
    - $a_x = 0$
    - $v_x = v_{0x}$
    - $x = x_0 + v_{0x}t$

  - y direction (free fall ... positive direction for y taken to be *upward*):

    (1) $v_y = v_{0y} - gt$

    (2) $y = y_0 + \frac{1}{2}\left(v_{0y} + v_y\right)t$

    (3) $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

    (4) $v_y^2 = v_{0y}^2 - 2g\left(y - y_0\right)$

- Relative Motion

  - $\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$

- Newton's Laws of Motion

  - Two broad categories of forces: *contact forces* (objects *in contact with one another*) and *field forces* (objects *not in contact with one another*). Gravity is the only field force we will deal with in this course.
  - Weight: $w = mg$
  - 1st law: $\vec{v}$ is constant unless object (or system) experiences net external force.
  - 2nd law :
    - $\sum \vec{F} = m\vec{a}$

    ∗implies three statements, in general: $\sum F_x = ma_x$, $\sum F_y = ma_y$, and $\sum F_z = ma_z$

    *for *systems* of objects, $\sum F_{ext} = m_{sys}a_{sys}$
  - 3rd law: Whenever one object exerts a force on a second object, the second exerts a force on the first; these two forces are equal in magnitude and opposite in direction:
    - $\vec{F}_{12} = -\vec{F}_{21}$

- Equilibrium

  - An object is said to be "in equilibrium" (really in "*translational*" equilibrium) if:
    - $\sum \vec{F} = \vec{0} \Rightarrow \vec{a} = \vec{0}$

      *really implies three *separate* requirements for translational equilibrium:

      1) $\sum F_x = 0 \Rightarrow a_x = 0$
      2) $\sum F_y = 0 \Rightarrow a_y = 0$
      3) $\sum F_z = 0 \Rightarrow a_z = 0$

- Friction forces

  - $f_s \le \mu_s n$
  - $f_k = \mu_k n$

Circular Motion

- 2nd law (centripetal direction): $\left(\sum F\right)_{rad} = ma_{rad}$
- Radial (centripetal) acceleration: $a_{rad} = \frac{v^2}{r}$
- If there is a tangential acceleration a_t also, then: $a_t = \frac{dv}{dt}$

Dot Product (Scalar Product)

- For any two vectors $\vec{A} = \langle A_x, A_y, A_z \rangle$ and $\vec{B} = \langle B_x, B_y, B_z \rangle$:
  - $\vec{A} \cdot \vec{B} \equiv AB\cos\theta$, in which:
    - $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$
    - $B = \sqrt{B_x^2 + B_y^2 + B_z^2}$
  - Alternative (equivalent) definition of dot product: $\vec{A} \cdot \vec{B} \equiv A_x B_x + A_y B_y + A_z B_z$

Work

- Work
  - Variable Forces:
    - $W \equiv \int \vec{F} \cdot d\vec{r}$ (general definition of work)
    - If $\vec{F}$ has only an *x* component and this component depends on *x*, then:
      $W = \int_{x_i}^{x_f} F(x)\,dx$
  - Constant Forces:
    - $W \equiv \vec{F} \cdot \Delta\vec{r}$ (2-D or 3-D path)
    - $W = F\Delta x$ (1-D path)

Springs

- Hooke's law: $\left|\vec{F}\right| = kx$. k = "the spring constant" or "the force constant".
- Potential energy stored in a spring (elastic potential energy): $U_{el} \equiv \frac{1}{2}kx^2$

Work-Energy Theorem

- $W_{net} = \Delta K$
- Kinetic energy: $K \equiv \frac{1}{2}mv^2$

Power

- Instantaneous Power:

- General Definition: Rate at which energy being supplied *by* or *to* a system: $P \equiv \frac{dE}{dt}$
- If energy comes from *work* being done, then the power is the rate at which work is done: $P = \frac{dW}{dt}$
- $P = \vec{F} \cdot \vec{v}$ (alternative definition)

- Average Power:
  - $P_{av} \equiv \frac{\Delta E}{\Delta t} = \frac{\Delta W}{\Delta t}$

Conservative Forces, Potential Energy, and Conservation of Energy

- Conservative Forces:
  - Work done by a conservative force: $W_c = -\Delta U$, for some potential energy U
  - Work done is independent of path.
  - Work done going once around *closed* path is zero.
- Potential Energy:
  - Gravitational potential energy (near surface of Earth): $U_{grav} \equiv mgy$
  - Elastic Potential Energy: $U_{el} \equiv \frac{1}{2}kx^2$
- Total mechanical energy: $E \equiv K + U$
- Nonconservative Forces: $W_{nc} = \Delta E$
- If $W_{nc} = 0$, E conserved.

Force and Potential Energy

- $F(x) = -\frac{dU(x)}{dx}$

Linear Momentum

- Linear momentum: $\vec{p} \equiv m\vec{v} \quad \Rightarrow \quad \begin{cases} p_x = mv_x \\ p_y = mv_y \\ p_z = mv_z \end{cases}$
- Newton's second law: $\vec{F}_{net} = \frac{d\vec{p}}{dt} \quad \Rightarrow \quad \left(\vec{F}_{net}\right)_{av} = \frac{\Delta\vec{p}}{\Delta t}$
  - $\vec{F}_{net}$ is *net (external) force* $\left(\vec{F}_{net} = \sum \vec{F}\right)$
- Impulse
  - Varying force: $\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}\, dt$ (general definition of impulse)
  - Constant force: $\vec{J} = \vec{F}\Delta t$
- Impulse-momentum theorem: $\vec{J}_{net} = \Delta\vec{p}$, in which:
  - $\vec{J}_{net} = \int_{t_i}^{t_f} \vec{F}_{net}\, dt$ (varying force) or $\vec{J}_{net} = \vec{F}_{net}\Delta t$ (constant force)

Collisions

- Two broad categories: *head-on* and *glancing*
- For each category, three classes:
  1. elastic: $\vec{p}$ and K conserved
  2. inelastic: $\vec{p}$ conserved, K not
  3. completely inelastic: $\vec{p}$ conserved, K not, objects stick together
- Head-on collisions:
  1. elastic:
     - $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ (p-conservation)
     - $v_{1i} + v_{1f} = v_{2i} + v_{2f}$ ("other" eq… derived in class)
  2. inelastic:
     - $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ (p-conservation)
  3. completely inelastic:
     - $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$ (p-conservation)
- Glancing (2-D) collisions:
  - $p_{ix} = p_{fx}$
  - $p_{iy} = p_{fy}$

Center of Mass and Systems of Particles

- $X_{\text{CM}} \equiv \dfrac{m_1 x_1 + m_2 x_2 + \cdots + m_N x_N}{m_1 + m_2 + \cdots + m_N} = \dfrac{m_1 x_1 + m_2 x_2 + \cdots + m_N x_N}{M_{tot}}$
- $Y_{\text{CM}} \equiv \dfrac{m_1 y_1 + m_2 y_2 + \cdots + m_N y_N}{m_1 + m_2 + \cdots + m_N} = \dfrac{m_1 y_1 + m_2 y_2 + \cdots + m_N y_N}{M_{tot}}$
- 2^{nd} law for system:
  - $\left(\sum \vec{F}\right)_{ext} = \dfrac{d\vec{P}}{dt}$, in which $\vec{P} \equiv m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_N\vec{v}_N$ is total momentum of system.
  - $\vec{P} = M_{tot}\vec{v}_{CM}$
  - If mass of system is constant, then:
    - $\left(\sum \vec{F}\right)_{ext} = M_{tot}\vec{a}_{\text{CM}}$

Rotational Kinematics

- angle θ in radians: $\theta \equiv \dfrac{s}{r}$ (s = length of arc swept out)
- angular displacement: $\Delta\theta \equiv \theta_f - \theta_i$
- average angular velocity: $\omega_{av} \equiv \dfrac{\Delta\theta}{\Delta t} = \dfrac{\theta_f - \theta_i}{t_f - t_i}$
- instantaneous angular velocity: $\omega \equiv \dfrac{d\theta}{dt}$
- average angular acceleration: $\alpha_{av} \equiv \dfrac{\Delta\omega}{\Delta t} = \dfrac{\omega_f - \omega_i}{t_f - t_i}$
- instantaneous angular acceleration: $\alpha \equiv \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

* θ understood to be in *radians* in all of the above

- Rotational motion with constant angular acceleration:
  (1) $\omega = \omega_0 + \alpha t$
  (2) $\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$
  (3) $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
  (4) $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Rotational and Linear Quantities

- $v_{\tan} = r\omega$
- $a_{\tan} = r\alpha$
- Radial (centripetal) acceleration: $a_{\text{rad}} = r\omega^2$

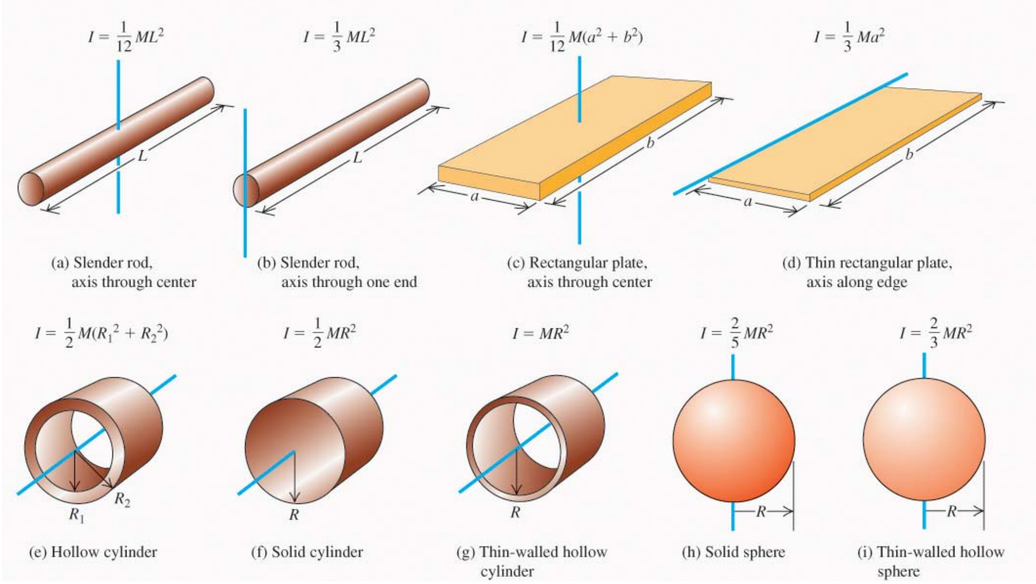
Rolling Without Slipping

- $v = R\omega$ (v = translational speed of center of rolling object)

Rotational Kinetic Energy and Moment of Inertia

- General Formula for Moment of Inertia: $I \equiv \int r^2\, dm$
- Moment of Inertia for Collection of N Point Masses: $I \equiv \sum_{i=1}^{N} m_i r_i^2$
- Moments of Inertia for Distributed Objects:

Table 9.2 Moments of Inertia of Various Bodies



- Rotational Kinetic Energy: $K_{rot} \equiv \frac{1}{2}I\omega^2$
- Total Kinetic Energy: $K_{total} = K_{trans} + K_{rot} = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$

(Note: Here v_{CM} is the translational velocity *of* the center of mass and I_{CM} is the moment of inertia *about* the center of mass.)

- Parallel-axis Theorem
  - $I_P = I_{CM} + Md^2$
- Perpendicular-axis Theorem
  - $I_z = I_x + I_y$
- Cross Product (Vector Product)
  - $\vec{A}\times\vec{B} = \left(A_yB_z - A_zB_y\right)\hat{x} + \left(A_zB_x - A_xB_z\right)\hat{y} + \left(A_xB_y - A_yB_x\right)\hat{z}$
  - $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi$
- Torque and Angular Acceleration
  - Torque: $\vec{\tau} \equiv \vec{r}\times\vec{F} = Fr_\perp$
  - Relation Between Torque and Angular Acceleration (Newton's Second Law for Rotational Motion):

$$\vec{\tau}_{net} = I\vec{\alpha}$$

- Angular Momentum
  - For point particle: $\vec{L} \equiv \vec{r}\times\vec{p}$.
  - For rigid body rotating about a fixed symmetry axis: $L = I\omega$.
  - Newton's Second Law for Rotational Motion: $\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$.

    (Note here that τ_{net} is the *net external* torque.)
- Equilibrium

Three conditions required for true equilibrium (translational *and rotational* equilibrium):

1. $\sum F_x = 0$ (i.e., $a_x = 0$)
2. $\sum F_y = 0$ (i.e., $a_y = 0$)
3. $\tau_{net} = 0$ for *any* axis (i.e., $\alpha = 0$ about *any* axis)

- Gravity
  - $F_{grav} = G\dfrac{m_1m_2}{r^2}$ (Newton's law of universal gravitation)
    - $G = 6.67\times10^{-11}\ \dfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}$ (gravitational constant)
  - $g = \dfrac{GM}{r^2}$ (acceleration due to gravity above planet, moon, etc., of mass *M*)
  - $U_{grav} = -G\dfrac{m_1m_2}{r}$ (gravitational potential energy of any two masses m_1 and m_2)
  - $v_{esc} = \sqrt{\dfrac{2GM}{R}}$ (escape velocity)

- Simple Harmonic Motion (SHM)
  - Relationships among frequency, angular frequency and period for sinusoids:
    - $f = \frac{1}{T}$
    - $\omega = 2\pi f$
    - $T = \frac{2\pi}{\omega}$
  - Mass on Spring
    - $x = A\cos(\omega t + \phi)$ (most general form of displacement as a function of time)
    - $\omega = \sqrt{\frac{k}{m}}$
    - $T = 2\pi\sqrt{\frac{m}{k}}$
    - $A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$
    - $\phi = \tan^{-1}\left(-\frac{v_0}{\omega x_0}\right)$
  - Simple Pendulum
    - $T = 2\pi\sqrt{\frac{L}{g}}$
  - Physical Pendulum
    - $T = 2\pi\sqrt{\frac{I}{mgd}}$
- Mechanical Waves
  - $v = \lambda f = \frac{\omega}{k}$ (phase velocity)
  - $y(x,t) = A\cos(kx - \omega t)$ (displacement of string for propagating wave)
  - $k = \frac{2\pi}{\lambda}$ (wave number)
  - $\frac{\partial^2 y(x,t)}{\partial x^2} = \left(\frac{1}{v^2}\right)\frac{\partial^2 y(x,t)}{\partial t^2}$ (1-D wave equation)
  - $v = \sqrt{\frac{F}{\mu}}$ (speed of transverse wave on string)
- Temperature Scales
  - $T_F = \frac{9}{5}T_C + 32$
  - $T_C = \frac{5}{9}(T_F - 32)$
  - $T_K = T_C + 273.15$

- Thermal Expansion
  - $\Delta L = \alpha L_0 \Delta T$
  - $\Delta A = 2\alpha A_0 \Delta T$
  - $\Delta V = \beta V_0 \Delta T = 3\alpha V_0 \Delta T$
- Calorimetry and Phase Changes
  - Heat you must put into a mass *m* of a substance to increase its temperature by ΔT :

$$Q = mc\Delta T$$

  - Heat you must put into (or take out of) a mass *m* of a substance to complete a phase change:

$$Q = mL$$

(L is the *latent heat* … of fusion, vaporization, etc.)

- Ideal Gas Law
  - $pV = NkT$
    - $p =$ pressure in pascals ($\mathrm{N/m^2}$)
    - $V =$ volume in $\mathrm{m^3}$
    - $N =$ number of particles
    - $k = 1.38\times10^{-23}$ J/K (Boltzmann's constant)
    - $T =$ temperature in kelvins

or:

  - $pV = nRT$
    - $p =$ pressure in pascals ($\mathrm{N/m^2}$) or atmospheres (atm)
    - $V =$ volume in $\mathrm{m^3}$ or L
    - $n =$ number of moles
    - $R = 8.31 \dfrac{\mathrm{J}}{\mathrm{mol\cdot K}}$ or $R = 0.0821 \dfrac{\mathrm{L\cdot atm}}{\mathrm{mol\cdot K}}$ (universal gas constant)
    - $T =$ temperature in kelvins
    - 1 atm = 1.013×10^5 Pa

- Work Done During Volume Changes
  - $W = \int p\,dV$ (work done by gas on its surroundings)
  - $W = p\Delta V$ (if p constant)

- First Law of Thermodynamics
  - $\Delta U = Q - W$
  - $dU = \underline{d}Q - \underline{d}W$
- Heat Engines and Refrigerators
  - $e = \dfrac{W}{Q_H} = 1 + \dfrac{Q_C}{Q_H}$
  - $K = \dfrac{|Q_C|}{|Q_H| - |Q_C|}$ (coefficient of performance of refrigerator)
  - $e_{Carnot} = 1 - \dfrac{T_C}{T_H}$ (efficiency of Carnot engine)

- Entropy

  - $dS = \frac{dQ}{T}$ (infinitesimal reversible process)
  - $\Delta S = \frac{Q}{T}$ (reversible isothermal process)
  - $\Delta S = \int_1^2 \frac{dQ}{T}$ (reversible process between states 1 and 2)

- Solar System Data

  - radius of Earth: $R_E = 6.37 \times 10^6$ m
  - radius of Moon: $R_M = 1.74 \times 10^6$ m
  - mass of Earth: $M_E = 5.97 \times 10^{24}$ kg
  - mass of Moon: 7.35×10^{22} kg
  - mass of Sun: 2.00×10^{30} kg
  - Earth-Moon distance (center-to-center): 3.84×10^8 m
  - Earth-Sun distance (center-to-center): 1.50×10^{11} m