Physics 2425
Formula Sheet – Exam 1

- ## One-dimensional kinematics

  - displacement: $\Delta x \equiv x_f - x_i$
  - $\text{average speed} \equiv \dfrac{\text{total distance traveled}}{\text{total time}}$
  - average velocity: $v_{av} \equiv \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$
  - instantaneous velocity: $v \equiv \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$
  - average acceleration: $a_{av} \equiv \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i}$
  - instantaneous acceleration: $a \equiv \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt} = \dfrac{d^2 x}{dt^2}$
  - One-dimensional motion with constant acceleration:
    (1) $v = v_0 + at$
    (2) $x = x_0 + \frac{1}{2}(v_0 + v)t$
    (3) $x = x_0 + v_0 t + \frac{1}{2}at^2$
    (4) $v^2 = v_0^2 + 2a(x - x_0)$

  - Free fall (positive direction for y taken to be *upward*)
    $*\ x \to y$ and $a \to -g$ in the above 4 equations of kinematics:

    (1) $v = v_0 - gt$
    (2) $y = y_0 + \frac{1}{2}(v_0 + v)t$
    (3) $y = y_0 + v_0 t - \frac{1}{2}gt^2$
    (4) $v^2 = v_0^2 - 2g(y - y_0)$

- ## Vectors

  ### Vectors in 2-D
  If a vector $\vec{A}$ is written in *component form* as $\vec{A} = A_x\hat{i} + A_y\hat{j} = \langle A_x, A_y \rangle$ then:

  - Getting magnitude and direction of $\vec{A}$ from the components:
    $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$ (magnitude of $\vec{A}$)
    $\theta = \arctan\left(\dfrac{A_y}{A_x}\right)$ (direction of $\vec{A}$)

- Getting components from magnitude and direction:

  $A_x = |\vec{A}|\cos\theta$ (x component of $\vec{A}$)

  $A_y = |\vec{A}|\sin\theta$ (y component of $\vec{A}$)

  } θ understood to be the angle that $\vec{A}$ makes with the positive x axis

Vectors in 3-D

If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = \langle A_x, A_y, A_z \rangle$ then:

- $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

2-D Kinematics

- position vector: $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} = \langle x(t), y(t) \rangle$
- average velocity: $\vec{v}_{av} \equiv \frac{\Delta\vec{r}}{\Delta t} = \left\langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t} \right\rangle$
- instantaneous velocity: $\vec{v} \equiv \frac{d\vec{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$
- average acceleration: $\vec{a}_{av} \equiv \frac{\Delta\vec{v}}{\Delta t} = \left\langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t} \right\rangle$
- instantaneous acceleration:
  - $\vec{a} \equiv \frac{d\vec{v}}{dt} = \left\langle \frac{dv_x}{dt}, \frac{dv_y}{dt} \right\rangle$
  - $\vec{a} \equiv \frac{d^2\vec{r}}{dt^2} = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$
- instantaneous speed $\equiv |\vec{v}|$ (magnitude of the instantaneous velocity)

3-D kinematics

- position vector: $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} = \langle x(t), y(t), z(t) \rangle$
- average velocity: $\vec{v}_{av} \equiv \frac{\Delta\vec{r}}{\Delta t} = \left\langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t} \right\rangle$
- instantaneous velocity: $\vec{v} \equiv \frac{d\vec{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$
- average acceleration: $\vec{a}_{av} \equiv \frac{\Delta\vec{v}}{\Delta t} = \left\langle \frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t} \right\rangle$
- instantaneous acceleration:
  - $\vec{a} \equiv \frac{d\vec{v}}{dt} = \left\langle \frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \right\rangle$
  - $\vec{a} \equiv \frac{d^2\vec{r}}{dt^2} = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \right\rangle$
- instantaneous speed $\equiv |\vec{v}|$ (magnitude of the instantaneous velocity)

Projectile Motion

- x direction (motion with constant velocity):
  - $a_x = 0$
  - $v_x = v_{0x}$
  - $x = x_0 + v_{0x}t$

- y direction (free fall ... positive direction for y taken to be *upward*):
  (1) $v_y = v_{0y} - gt$
  (2) $y = y_0 + \frac{1}{2}\left(v_{0y} + v_y\right)t$
  (3) $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$
  (4) $v_y^2 = v_{0y}^2 - 2g\left(y - y_0\right)$

Relative Motion

- $\vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB}$

Newton's Laws of Motion

- Weight: $w = mg$
- 1st law: $\vec{v}$ is constant unless object experiences net external force.
- 2nd law :
  - $\sum \vec{F} = m\vec{a}$

  *implies three statements, in general: $\sum F_x = ma_x$, $\sum F_y = ma_y$, and $\sum F_z = ma_z$
- 3rd law: Whenever one object exerts a force on a second object, the second exerts a force on the first; these two forces are equal in magnitude and opposite in direction:
  - $\vec{F}_{12} = -\vec{F}_{21}$

Equilibrium

- An object is said to be "in equilibrium" (really in "*translational*" equilibrium) if:
  - $\sum \vec{F} = \vec{0} \Rightarrow \vec{a} = \vec{0}$

    *really implies three *separate* requirements for translational equilibrium:

    1) $\sum F_x = 0 \Rightarrow a_x = 0$
    2) $\sum F_y = 0 \Rightarrow a_y = 0$
    3) $\sum F_z = 0 \Rightarrow a_z = 0$

Circular Motion

- Radial (centripetal) acceleration: $a_{rad} = a_\perp = \frac{v^2}{r}$
- If there is a tangential acceleration $a_{||}$ also, then: $a_{||} = \frac{dv}{dt}$
- 2nd law (centripetal direction): $\left(\sum F\right)_{rad} = ma_{rad}$