Chapter 2: One-Dimensional Kinematics

kinematics: study of motion without regard to what *caused* it

One-dimensional motion is motion along a **line**.

Three Key Concepts

1. displacement
2. velocity
3. acceleration

Displacement

$$\Delta x \equiv x_f - x_i \tag{1}$$

Notes:

1. can be + or –
   - sign indicates direction of motion:
     - if Δx is $+$, motion is in $+x$ direction
     - if Δx is $-$, motion is in $-x$ direction

2. can be zero (for any “*round trip*”)

3. not same as distance traveled!

Velocity

Two kinds:

Average:

$$v_{av} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \tag{2}$$

Tells how rapidly (on *average*, over the *whole* Δt) the position is *changing*.

Instantaneous:

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{3}$$

Tells how rapidly the position is changing “right now,” at some *instant* in time.

Average Velocity

- sign (+/-) is same as sign of Δx, with same interpretation:
  - o if v_{av} is +, motion is in the $+x$ direction
  - o if v_{av} is -, motion is in the $-x$ direction
- is zero for any “round trip”
- not same as average speed!
- unit (SI): m/s

Graphical Interpretation of Average Velocity

On a graph of x vs. t, v_{av} represents the **slope of the secant line** (line joining the “initial” and “final” points)

Graphical Interpretation of Instantaneous Velocity

On a graph of x vs. t, v represents the **slope of the tangent line.**

Acceleration

Two kinds:

Average:

$$a_{av} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad (4)$$

Tells how rapidly (on *average*, over the *whole* Δt) the *velocity* is changing.

Instantaneous:

$$a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2} \qquad (5)$$

Tells how rapidly the velocity is changing “right now,” at some *instant* in time.

Average Acceleration

- unit (SI): m/s^2
- can be $+$ or $-$, but **be careful**:
  - o a *positive* a_{av} **does not necessarily mean** object is *speeding up*!
  - o a *negative* a_{av} does not necessarily mean object is *slowing down*!
- is zero whenever $v_f = v_i$ (regardless of what happened *between* t_i and t_f).

Graphical Interpretation of Average Acceleration

On a graph of v vs. t, a_{av} represents the **slope of the secant line** (line joining the “initial” and “final” points)

Graphical Interpretation of Instantaneous Acceleration

On a graph of v vs. t, a represents the **slope of the tangent line.**

One-Dimensional Motion With Constant Acceleration

For straight-line motion with *constant* acceleration, it's possible to derive four very useful equations relating position, velocity, acceleration, and time.

key idea: Whenever a is *constant*, $a = a_{av}$.

Equations of Kinematics for
1-D Motion w/Constant Acceleration

Recall the definition of a_{av}:

$$a_{av} \equiv \frac{v_f - v_i}{t_f - t_i}$$

If the object moves with constant a, then:

$$a = \frac{v_f - v_i}{t_f - t_i}$$

If we now make the following choices:

- let $t_i = 0$ (Always start timing when stopwatch reads *zero*.)
- call "t_f" just "t" (any time *after* $t = 0$)
- "v_i" → "v_0" (pronounced "vee-naught"…velocity at time $t = 0$)
- call "v_f" just "v" (velocity at the later time t)

then:

$$a = \frac{v - v_0}{t}$$

Rearrange and we get the first of the four equations of kinematics:

$$v = v_0 + at \tag{6}$$

To get the second one, start with the definition of v_{av}:

$$v_{av} \equiv \frac{x_f - x_i}{t_f - t_i}$$

Now make a set of choices of notation exactly similar to what we did in deriving (6):

- let $t_i = 0$
- call "t_f" just "t" (any time *after* $t = 0$)
- "x_i" → "x_0" (position at time $t = 0$)
- call "x_f" just "x" (position at the later time t)

With these choices, we have:

$$v_{av} \equiv \frac{x - x_0}{t}$$

$$x = x_0 + v_{av} t$$

If the acceleration is *constant*, then v_{av} is just the arithmetic *mean* of the initial and final velocities:

$$v_{av} = \frac{v_0 + v}{2}$$

So:

$$x = x_0 + \frac{1}{2}\left(v_0 + v\right)t \tag{7}$$

The third equation of kinematics follows at once from plugging (6) into (7):

$$x = x_0 + \frac{1}{2}\left(v_0 + v_0 + at\right)t$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{8}$$

Finally, the fourth equation of kinematics for 1-D motion with constant acceleration follows at once from rearranging (6) for t and plugging into (7):

$$x = x_0 + \frac{1}{2}\left(v_0 + v\right)\left(\frac{v - v_0}{a}\right)$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \tag{9}$$

Eqs. 6-9 are extremely useful in solving problems involving things that move in straight lines with constant acceleration.

Free-Fall

An example of 1-D motion with constant acceleration.

An object is said to be in free-fall if essentially the only force acting on it is the force of **gravity** (air resistance *negligible*).

Galileo Galilei (1564-1642) discovered that all objects in free-fall move with a constant acceleration of magnitude:

$$g = 32\ \mathrm{ft/s^2} = 9.81\ \mathrm{m/s^2} \tag{10}$$

This is referred to as *the acceleration due to gravity* or *the free-fall acceleration*.

Note that the symbol g is reserved for **positive** $9.81\ \mathrm{m/s^2}$, **never negative**.

Typically, we will measure the vertical position by a coordinate y, with $+y$ chosen to be **upward**. Then Eqs. 6-9 are still true, but with $x \to y$ and $a \to -g$:

$$v = v_0 - gt \tag{11}$$

$$y = y_0 + \frac{1}{2}\left(v_0 + v\right)t \tag{12}$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \tag{13}$$

$$v^2 = v_0^2 - 2g\left(y - y_0\right) \tag{14}$$

Derivatives

Consider some function $f(t)$. The variable t represents time. The first derivative of f with respect to t, which is written df/dt, is defined as follows:

$$\frac{df}{dt} \equiv \lim_{\Delta t \to 0} \left[\frac{f(t+\Delta t) - f(t)}{\Delta t} \right]$$

On a graph of $f(t)$ versus t, df/dt represents the slope of the tangent line. If we evaluate df/dt at a particular point on the graph (at $t = c$, e.g.), we get the slope of the tangent line at $t = c$.

Rule for Differentiating Polynomial Functions

Differentiating a function means finding the derivative of the function.

For now, the only kind of function you will be asked to differentiate is a polynomial function, i.e., a function $f(t)$ of the form:

$$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots + a_n t^n,$$

in which the number *n* gives the order of the polynomial.

Consider a polynomial consisting of just a single term:

$$f(t) = a_n t^n$$

Rule for Differentiating Polynomial Terms:

- Bring down the exponent as a coefficient.
- Decrease the exponent by 1.

So:

$$\frac{df}{dt} = \frac{d}{dt}\left(a_n t^n\right) = n a_n t^{n-1}$$

Two Important Properties of the Derivative

1. For any constant a,

$$\frac{d}{dt}\left[af(t)\right]=a\frac{d}{dt}\left[f(t)\right]$$

2. The derivative of a sum of functions is the sum of the individual derivatives:

$$\frac{d}{dt}\left[f(t)+g(t)\right]=\frac{d}{dt}\left[f(t)\right]+\frac{d}{dt}\left[g(t)\right]$$

The above two properties imply that to differentiate a general polynomial, we can just differentiate term-by-term and add the results.

For the most general polynomial,

$$f\left(t\right)=a_0+a_1t+a_2t^2+a_3t^3+\cdots+a_nt^n\text{,}$$

the derivative is:

$$\frac{df}{dt}=\frac{d}{dt}\left[f\left(t\right)\right]=a_1+2a_2t+3a_3t^2+\cdots+na_nt^{n-1}\text{.}$$