Physics 1401
Formula Sheet – Final

One-dimensional kinematics

- displacement: $\Delta x \equiv x_f - x_i$
- $\text{average speed} \equiv \dfrac{\text{total distance traveled}}{\text{total time}}$
- $\text{instantaneous speed} \equiv |\vec{v}|$ (magnitude of the instantaneous velocity)
- average velocity: $v_{av} \equiv \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$
- instantaneous velocity: $v \equiv \lim\limits_{\Delta t \to 0} v_{av} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t}$
- average acceleration: $a_{av} \equiv \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i}$
- instantaneous acceleration: $a \equiv \lim\limits_{\Delta t \to 0} a_{av} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t}$
- One-dimensional motion with constant acceleration (4 facts):
  (1) $v = v_0 + at$
  (2) $x = x_0 + \frac{1}{2}(v_0 + v)t$
  (3) $x = x_0 + v_0 t + \frac{1}{2}at^2$
  (4) $v^2 = v_0^2 + 2a(x - x_0)$

- Free fall (positive direction for y taken to be *upward*)
  $* \; x \to y$ and $a \to -g$ in the above 4 facts:

  (1) $v = v_0 - gt$
  (2) $y = y_0 + \frac{1}{2}(v_0 + v)t$
  (3) $y = y_0 + v_0 t - \frac{1}{2}gt^2$
  (4) $v^2 = v_0^2 - 2g(y - y_0)$

Vectors

The *resultant vector* for several vectors is the *vector sum*. For example, if you have three displacements $\vec{d}_1$, $\vec{d}_2$, and $\vec{d}_3$, the *resultant* displacement $\vec{R}$ is given by:

$$\vec{R} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 .$$

If a vector $\vec{A}$ is written in *component form* as $\vec{A} = A_x\hat{x} + A_y\hat{y}$ then:

- Getting magnitude and direction of $\vec{A}$ from the components:
  $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$ (magnitude of $\vec{A}$)
  $\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$ (direction of $\vec{A}$)

- Getting components from magnitude and direction:

  $A_x = |\vec{A}|\cos\theta$ (x component of $\vec{A}$)

  $A_y = |\vec{A}|\sin\theta$ (y component of $\vec{A}$)

  } θ understood to be the angle that $\vec{A}$ makes with the positive x axis

Projectile Motion

- x direction (motion with constant velocity):
  - $a_x = 0$
  - $v_x = v_{x0}$
  - $x = x_0 + v_{x0}t$

- y direction (free fall ... positive direction for y taken to be *upward*):

  (1) $v_y = v_{y0} - gt$

  (2) $y = y_0 + \frac{1}{2}(v_{y0} + v_y)t$

  (3) $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$

  (4) $v_y^2 = v_{y0}^2 - 2g(y - y_0)$

Newton's Laws of Motion

- Two kinds of forces: *contact forces* (objects *in contact with one another*) and *field forces* (objects *not in contact with one another*). Gravity is the only field force we will deal with in this course.
- $\sum \vec{F} = m\vec{a}$ (Newton's 2nd law)

  ∗implies two statements: $\sum F_x = ma_x$ and $\sum F_y = ma_y$
- Weight: $W = mg$
- Normal forces: in general, whenever two surfaces are in contact, each surface exerts a force on the other in a direction that is perpendicular to the two surfaces.

Friction forces

- Two kinds: the *force of static friction* , f_s and the *force of kinetic friction*, f_k :
  - $f_s \le \mu_s N$ (force of static friction)
  - $f_k = \mu_k N$ (force of kinetic friction)
- $\mu_s =$ "the coefficient of static friction"; $\mu_k =$ "the coefficient of kinetic friction"

Strings and Springs

- Strings
  - the *tension* in a string *always pulls* on the objects attached to the ends of the string
- Springs
  - Hooke's law: $|\vec{F}| = kx$. k = "the spring constant" or "the force constant".
  - Work required to stretch or compress a spring a distance x from its equilibrium length:

    $$W = \frac{1}{2}kx^2$$

  - Potential energy stored in a spring: $U_{spring} \equiv \frac{1}{2}kx^2$

- Translational Equilibrium

  - $\sum \vec{F} = \vec{0} \Rightarrow \vec{a} = \vec{0}$.
  - The statement immediately above really implies two sets of statements: $\sum F_x = 0$ and $\sum F_y = 0 \quad \Rightarrow \quad a_x = 0$ and $a_y = 0$.

- Circular Motion

  - Centripetal acceleration: $a_{cp} = \dfrac{v^2}{r}$

- Work and Kinetic Energy

  - Work: $W \equiv Fd\cos\theta$
  - Kinetic energy: $K \equiv \dfrac{1}{2}mv^2$
  - Work-energy theorem: $W_{net} = \Delta K$
  - Work required to stretch or compress a spring a distance x from its equilibrium length:
  $$W = \frac{1}{2}kx^2$$

- Potential Energy and Conservative Forces

  - Work done by a conservative force: $W_c = -\Delta U$, for some potential energy U
  - Gravitational potential energy (near surface of Earth): $U_{grav} \equiv mgy$
  - Potential energy stored in a spring: $U_{spring} \equiv \dfrac{1}{2}kx^2$
  - Total mechanical energy: $E \equiv K + U$

- Linear Momentum

  - Linear momentum: $\vec{p} \equiv m\vec{v}$
  - Newton's second law:
  $$\sum \vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}$$
  (instantaneous net force equals instantaneous rate of change of linear momentum). Implies the following:
  $$\left(\sum \vec{F}\right)_{av} = \frac{\Delta \vec{p}}{\Delta t}$$
  (*average* net force equals *average* rate of change of linear momentum).
  - Impulse: $\vec{I} \equiv \left(\sum \vec{F}\right)_{av} \Delta t$
  - Impulse-momentum theorem: $\vec{I} = \Delta \vec{p}$

Collisions

- Two broad categories: *head-on* and *glancing*
- For each category, three classes:
  1. elastic: $\vec{p}$ and K conserved
  2. inelastic: $\vec{p}$ conserved, K not
  3. completely inelastic: $\vec{p}$ conserved, K not, objects stick together
- Head-on collisions:
  1. elastic:
     - $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ (p-conservation)
     - $v_{1i} + v_{1f} = v_{2i} + v_{2f}$ ("other" equation… derived in class)
  2. inelastic:
     - $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ (p-conservation)
  3. completely inelastic:
     - $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$ (p-conservation)
- Glancing collisions:
  - $p_{xi} = p_{xf}$
  - $p_{yi} = p_{yf}$

Rotational Kinematics

- angle θ in radians: $\theta \equiv \dfrac{s}{r}$ (s = length of arc swept out)
- angular displacement: $\Delta\theta \equiv \theta_f - \theta_i$
- average angular velocity: $\omega_{av} \equiv \dfrac{\Delta\theta}{\Delta t} = \dfrac{\theta_f - \theta_i}{t_f - t_i}$
- instantaneous angular velocity: $\omega \equiv \lim\limits_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t}$
- average angular acceleration: $\alpha_{av} \equiv \dfrac{\Delta\omega}{\Delta t} = \dfrac{\omega_f - \omega_i}{t_f - t_i}$
- instantaneous angular acceleration: $\alpha \equiv \lim\limits_{\Delta t \to 0} \dfrac{\Delta\omega}{\Delta t}$

  * θ understood to be in *radians* in all of the above

- Period (time required for one complete revolution): $T = \dfrac{2\pi}{\omega}$
- Rotational motion with constant angular acceleration (4 facts):
  (1) $\omega = \omega_0 + \alpha t$
  (2) $\theta = \theta_0 + \dfrac{1}{2}(\omega_0 + \omega)t$
  (3) $\theta = \theta_0 + \omega_0 t + \dfrac{1}{2}\alpha t^2$
  (4) $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Rotational and Linear Quantities

- $v_t = r\omega$
- $a_t = r\alpha$
- Centripetal acceleration: $a_{cp} = r\omega^2$
- *rigid* object: Every point in the object has the same angular velocity as every other point in the object. Every point in the object has the same angular acceleration as every other point in the object.

Rolling Without Slipping

- $v = r\omega$ (v = translational speed of center of rolling object)
- $v_{\text{point of contact}} = 0$ at each instant

Rotational Kinetic Energy and Moment of Inertia

- General Formula for Moment of Inertia for Collection of *N* Point Masses: $I \equiv \sum_{i=1}^{N} m_i r_i^2$
- Moments of Inertia for Distributed Objects: Table 10-1 will be given.
- Rotational Kinetic Energy: $K_{rot} \equiv \frac{1}{2} I\omega^2$
- Total Kinetic Energy: $K_{total} = K_{trans} + K_{rot} = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_{CM}\,\omega^2$

(Note: Here v_{CM} is the translational velocity *of* the center of mass and I_{CM} is the moment of inertia *about* the center of mass.)

Torque and Angular Acceleration

- Torque: $\tau \equiv rF$ (*r* is the moment arm)
- Relation Between Torque and Angular Acceleration (Newton's Second Law for Rotational Motion):

$$\tau_{net} = I\alpha$$

Equilibrium

Three conditions required for true equilibrium (translational *and rotational* equilibrium):

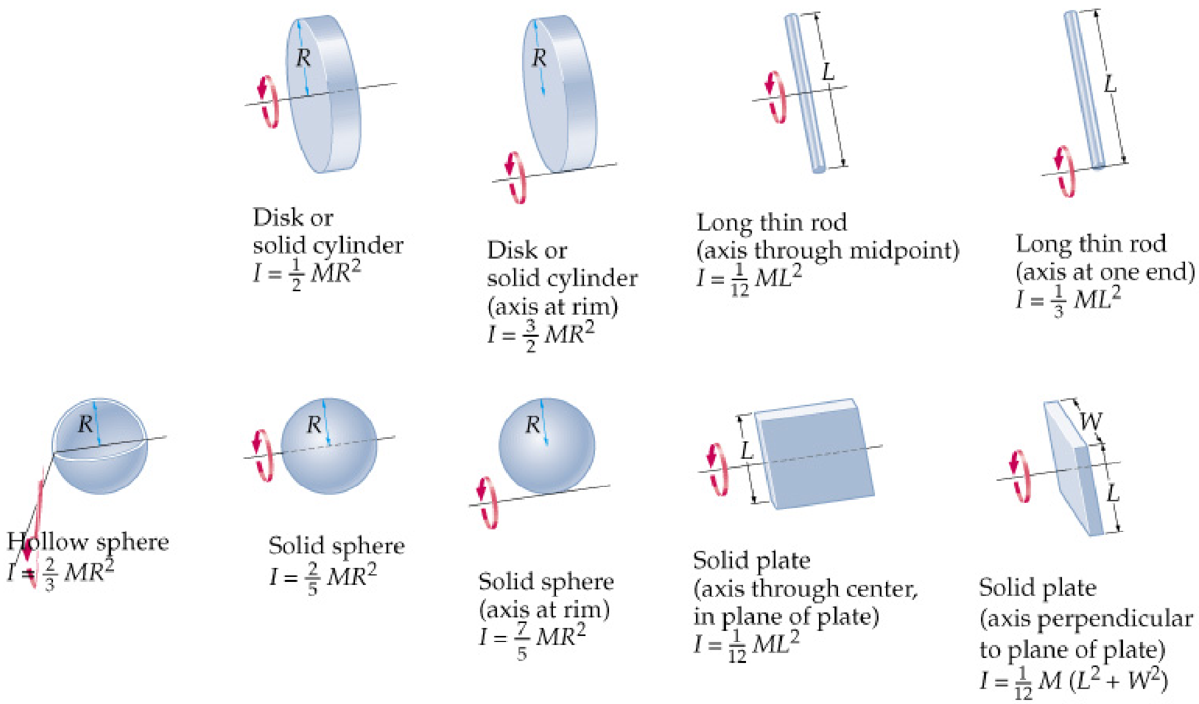
1. $\sum F_x = 0$ (i.e., $a_x = 0$)
2. $\sum F_y = 0$ (i.e., $a_y = 0$)
3. $\tau_{net} = 0$ for *any* axis (i.e., $\alpha = 0$ about *any* axis)

Angular Momentum

- Angular momentum: $L \equiv I\omega$.
- For point particle of mass m moving with speed v at an angle θ with respect to the radial direction: $L = rmv\sin\theta$.
- Newton's Second Law for Rotational Motion: $\tau_{net} = \dfrac{\Delta L}{\Delta t}$.

  Note here that τ_{net} is the *average net external* torque.

Table 10-1: Moments of Inertia for Uniform, Rigid Objects of Various Shapes



Gravity

- $F_{grav} = G\dfrac{m_1 m_2}{r^2}$ (Newton's law of universal gravitation)
  - $G = 6.67 \times 10^{-11}\ \dfrac{\mathrm{N \cdot m^2}}{\mathrm{kg}^2}$ (gravitational constant)
- $U_{grav} = -G\dfrac{m_1 m_2}{r}$ (gravitational potential energy of any two masses m_1 and m_2)
  - for mass m a distance r from center of Earth: $U_{grav} = -G\dfrac{mM_E}{r}$
- escape speed (on Earth): $v_e = \sqrt{\dfrac{2GM_E}{R_E}}$

- Oscillations About Equilibrium
  - Relationships among frequency, angular frequency and period for sinusoids:
    - $f = \frac{1}{T}$
    - $\omega = 2\pi f$
    - $T = \frac{2\pi}{\omega}$
  - Mass on Spring
    - $x = A\cos(\omega t + \phi)$ (most general form of displacement as a function of time)
    - $\omega = \sqrt{\frac{k}{m}}$
    - $T = 2\pi\sqrt{\frac{m}{k}}$
  - Simple Pendulum
    - $T = 2\pi\sqrt{\frac{L}{g}}$

- Waves and Sound
  - $v = \lambda f$
  - $v = 343$ m/s (speed of sound in air at standard temperature and pressure)
  - Doppler Effect
    - Moving observer: $f' = f\left(1 \pm \frac{u}{v}\right)$
    - Moving source: $f' = f\left(\frac{1}{1 \mp \frac{u}{v}}\right)$

- Temperature Scales
  - $T_F = \frac{9}{5}T_C + 32$
  - $T_C = \frac{5}{9}(T_F - 32)$
  - $T_K = T_C + 273.15$

- Heat and Mechanical Work
  - "Mechanical Equivalent of Heat": $1\,\text{cal} = 4.186\,\text{J}$

- Specific Heat
  - Heat you must put into a mass m of a substance to increase its temperature by ΔT :

$$Q = mc\Delta T$$

c is the specific heat of the substance.

- Ideal Gas Law

  - $PV = NkT$
    - $P =$ pressure in pascals ($\mathrm{N/m^2}$)
    - $V =$ volume in $\mathrm{m^3}$
    - $N =$ number of particles
    - $k = 1.38\times10^{-23}$ J/K (Boltzmann's constant)
    - $T =$ temperature in kelvins

  or:
  - $PV = nRT$
    - $P =$ pressure in pascals ($\mathrm{N/m^2}$)
    - $V =$ volume in $\mathrm{m^3}$
    - $n =$ number of moles
    - $R = 8.31 \dfrac{\mathrm{J}}{\mathrm{mol\cdot K}}$ (universal gas constant)
    - $T =$ temperature in kelvins

- Latent Heats

  - Heat you must put into (or take out of) a mass m of a substance to complete a phase change (with no change in temperature):

$$Q = mL$$

  (L is the *latent heat* … of fusion, vaporization, etc.)

- First Law of Thermodynamics

  - $\Delta U = Q - W$

- Thermal Processes

  - $W = P\Delta V$ (work done in constant-pressure expansion or compression)
  - $W = NkT \ln\left(\dfrac{V_f}{V_i}\right)$ (work done in isothermal expansion or compression)

- Heat Engines and the Carnot Cycle

  - $e = 1 - \dfrac{Q_c}{Q_h}$
  - $e_{\max} = 1 - \dfrac{T_c}{T_h}$
  - $W_{\max} = \left(1 - \dfrac{T_c}{T_h}\right)Q_h$

- Entropy

  - $\Delta S = \dfrac{Q}{T}$

- Solar System Data

  - radius of Earth: $R_E = 6.37\times10^6$ m
  - radius of Moon: $R_M = 1.74\times10^6$ m
  - mass of Earth: $M_E = 5.97\times10^{24}$ kg
  - mass of Moon: 7.35×10^{22} kg
  - mass of Sun: 2.00×10^{30} kg
  - Earth-Moon distance: 3.84×10^8 m
  - Earth-Sun distance: 1.50×10^{11} m