Physics 1401
Formula Sheet – Exam 2

▪ One-dimensional kinematics

- displacement: $\Delta x \equiv x_f - x_i$
- $\text{average speed} \equiv \dfrac{\text{total distance traveled}}{\text{total time}}$
- $\text{instantaneous speed} \equiv |\vec{v}|$ (magnitude of the instantaneous velocity)
- average velocity: $v_{av} \equiv \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$
- instantaneous velocity: $v \equiv \lim_{\Delta t \to 0} v_{av} = \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t}$
- average acceleration: $a_{av} \equiv \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i}$
- instantaneous acceleration: $a \equiv \lim_{\Delta t \to 0} a_{av} = \lim_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t}$
- One-dimensional motion with constant acceleration (4 facts):
  (1) $v = v_0 + at$
  (2) $x = x_0 + \frac{1}{2}(v_0 + v)t$
  (3) $x = x_0 + v_0 t + \frac{1}{2}at^2$
  (4) $v^2 = v_0^2 + 2a(x - x_0)$
- Free fall (positive direction for y taken to be *upward*)
  $* \; x \to y$ and $a \to -g$ in the above 4 facts:

  (1) $v = v_0 - gt$
  (2) $y = y_0 + \frac{1}{2}(v_0 + v)t$
  (3) $y = y_0 + v_0 t - \frac{1}{2}gt^2$
  (4) $v^2 = v_0^2 - 2g(y - y_0)$

▪ Vectors

The *resultant vector* for several vectors is the *vector sum*. For example, if you have three displacements $\vec{d}_1$, $\vec{d}_2$, and $\vec{d}_3$, the *resultant* displacement $\vec{R}$ is given by:

$$\vec{R} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 .$$

If a vector $\vec{A}$ is written in *component form* as $\vec{A} = A_x\hat{x} + A_y\hat{y}$ then:

- Getting magnitude and direction of $\vec{A}$ from the components:
  $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$ (magnitude of $\vec{A}$)
  $\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$ (direction of $\vec{A}$)

- Getting components from magnitude and direction:

  $A_x = |\vec{A}|\cos\theta$ (x component of $\vec{A}$)

  $A_y = |\vec{A}|\sin\theta$ (y component of $\vec{A}$)

  } θ understood to be the angle that $\vec{A}$ makes with the positive x axis

Projectile Motion

- x direction (motion with constant velocity):
  - $a_x = 0$
  - $v_x = v_{x0}$
  - $x = x_0 + v_{x0}t$

- y direction (free fall ... positive direction for y taken to be *upward*):

  (1) $v_y = v_{y0} - gt$

  (2) $y = y_0 + \frac{1}{2}(v_{y0} + v_y)t$

  (3) $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$

  (4) $v_y^2 = v_{y0}^2 - 2g(y - y_0)$

Newton's Laws of Motion

- Two kinds of forces: *contact forces* (objects *in contact with one another*) and *field forces* (objects *not in contact with one another*). Gravity is the only field force we will deal with in this course.
- $\sum \vec{F} = m\vec{a}$ (Newton's 2nd law)

  ∗implies two statements: $\sum F_x = ma_x$ and $\sum F_y = ma_y$
- Weight: $W = mg$
- Normal forces: in general, whenever two surfaces are in contact, each surface exerts a force on the other in a direction that is perpendicular to the two surfaces.

Friction forces

- Two kinds: the *force of static friction* , f_s and the *force of kinetic friction*, f_k :
  - $f_s \le \mu_s N$ (force of static friction)
  - $f_k = \mu_k N$ (force of kinetic friction)
- $\mu_s =$ "the coefficient of static friction"; $\mu_k =$ "the coefficient of kinetic friction"

Strings and Springs

- Strings
  - the *tension* in a string *always pulls* on the objects attached to the ends of the string
- Springs
  - Hooke's law: $|\vec{F}| = kx$. k = "the spring constant" or "the force constant".
  - Work required to stretch or compress a spring a distance x from its equilibrium length:

    $$W = \frac{1}{2}kx^2$$

  - Potential energy stored in a spring: $U_{spring} \equiv \frac{1}{2}kx^2$

- Translational Equilibrium
  - $\sum \vec{F} = \vec{0} \;\Rightarrow\; \vec{a} = \vec{0}$.
  - The statement immediately above really implies two sets of statements:
    $\sum F_x = 0$ and $\sum F_y = 0 \quad \Rightarrow \quad a_x = 0$ and $a_y = 0$.

- Circular Motion
  - Centripetal acceleration: $a_{cp} = \dfrac{v^2}{r}$

- Work and Kinetic Energy
  - Work: $W \equiv Fd\cos\theta$
  - Kinetic energy: $K \equiv \dfrac{1}{2}mv^2$
  - Work-energy theorem: $W_{net} = \Delta K$
  - Work required to stretch or compress a spring a distance x from its equilibrium length:

$$W = \frac{1}{2}kx^2$$

- Potential Energy and Conservative Forces
  - Work done by a conservative force: $W_c = -\Delta U$, for some potential energy U
  - Gravitational potential energy (near surface of Earth): $U_{grav} \equiv mgy$
  - Potential energy stored in a spring: $U_{spring} \equiv \dfrac{1}{2}kx^2$
  - Total mechanical energy: $E \equiv K + U$

- Linear Momentum
  - Linear momentum: $\vec{p} \equiv m\vec{v}$
  - Newton's second law:

$$\sum \vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}$$

(instantaneous net force equals instantaneous rate of change of linear momentum). Implies the following:

$$\left(\sum \vec{F}\right)_{av} = \frac{\Delta \vec{p}}{\Delta t}$$

(*average* net force equals *average* rate of change of linear momentum).

  - Impulse: $\vec{I} \equiv \left(\sum \vec{F}\right)_{av} \Delta t$
  - Impulse-momentum theorem: $\vec{I} = \Delta \vec{p}$

- Collisions

  - Two broad categories: *head-on* and *glancing*
  - For each category, three classes:
    1. elastic: $\vec{p}$ and K conserved
    2. inelastic: $\vec{p}$ conserved, K not
    3. completely inelastic: $\vec{p}$ conserved, K not, objects stick together
  - Head-on collisions:
    1. elastic:
       - $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ (p-conservation)
       - $v_{1i} + v_{1f} = v_{2i} + v_{2f}$
    2. inelastic:
       - $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ (p-conservation)
    3. completely inelastic:
       - $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$ (p-conservation)
  - Glancing collisions:
    - $p_{xi} = p_{xf}$
    - $p_{yi} = p_{yf}$