Physics 1401
Homework Solutions - Walker, Chapter 12

Conceptual Questions

CQ1. No, it is not correct. For an astronaut's weight to be zero—that is, for him to be "beyond the pull of Earth's gravity"—he would have to be *infinitely* far away from the earth. In fact, at a typical orbital height (for a Space Shuttle mission, for example) of 150 mi, or 241,000 m, the astronaut experiences a free-fall acceleration of:

$$g_{\text{in orbit}} = \frac{GM_E}{r^2} = \frac{\left(6.673\times10^{-11}\right)\left(5.97\times10^{24}\right)}{\left(241{,}000\text{ m} + 6.37\times10^{6}\text{ m}\right)^2} = 9.12\text{ m/s}^2 .$$

Note that in the equation above, *r* represents the distance from the Space Shuttle to the *center* of the earth... So $r = h + R_E$.

The value for *g* just derived is just a little bit less than *g* at the surface of the earth. For an astronaut of typical mass ($m \approx 70$ kg), the weight of this astronaut while orbiting aboard the Space Shuttle is:

$$w_{\text{in orbit}} = mg_{\text{in orbit}} = (70\text{ kg})\left(9.12\text{ m/s}^2\right) \approx 640\text{ N}\text{, or about 140 lb.}$$

So the astronaut is definitely *not* weightless. But he *feels* weightless because he and the Space Shuttle are in *free-fall* with the *same* acceleration, 9.12 m/s^2 . For an astronaut standing on the floor of the Space Shuttle, Newton's second law says:

$$w - N = ma .$$

So the normal force is: $N = w - ma = mg_{\text{in orbit}} - ma$.

But since the astronaut is in *free-fall*, his acceleration *a* is equal to $g_{\text{in orbit}}$. The result is that *N* = 0... The floor exerts a force of *zero newtons* on the astronaut. It is *as though* the astronaut were standing on the surface of the earth, but the force of gravity had been "turned off." (Although this is not *really* the case, of course.) It is in this sense that the astronaut feels "weightless."

Problems

8. (a.) Consider picture below. The Sun pulls on the Earth with the gravitational force shown as F_{SE} in the figure. The Moon pulls on the Earth with the gravitational force shown as F_{ME} .

With the coordinate system chosen as shown in the figure, the force $\vec{F}_{SE}$ is given by:

$$\vec{F}_{SE} = G\frac{M_S M_E}{R_{SE}^2}\hat{x} ,$$

in which R_{SE} is the Sun-Earth distance. And, similarly, $\vec{F}_{ME}$ is given by:

$$\vec{F}_{ME} = G\frac{M_M M_E}{R_{ME}^2}\hat{y},$$

in which R_{ME} is the Moon-Earth distance. So the *net* force on the Earth is:

$$\vec{F}_{net} = \vec{F}_{SE} + \vec{F}_{ME}$$

$$\vec{F}_{net} = G\frac{M_S M_E}{R_{SE}^2}\hat{x} + G\frac{M_M M_E}{R_{ME}^2}\hat{y}$$

Plugging in numbers:

$$\vec{F}_{net} = \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(2.00\times10^{30}\ \text{kg}\right)\left(5.97\times10^{24}\ \text{kg}\right)}{\left(1.50\times10^{11}\ \text{m}\right)^2}\hat{x} + \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(7.35\times10^{22}\ \text{kg}\right)\left(5.97\times10^{24}\ \text{kg}\right)}{\left(3.84\times10^{8}\ \text{m}\right)^2}\hat{y}$$

$$\vec{F}_{net} = \left(3.54\times10^{22}\ \text{N}\right)\hat{x} + \left(1.98\times10^{20}\ \text{N}\right)\hat{y}$$

Notice that the force that the Sun exerts on the Earth is more than 100 times larger than the force the Moon exerts on the Earth, even though the Moon is much closer! The magnitude of the net force is:

$$F_{net} = \sqrt{\left(3.54\times10^{22}\ \text{N}\right)^2 + \left(1.98\times10^{20}\ \text{N}\right)^2} = 3.54\times10^{22}\ \text{N},$$

which is essentially just the force that the Sun exerts on the Earth.

(b.) To find the direction, I can find the angle that $\vec{F}_{net}$ makes with the positive *x* axis:

$$\theta = \tan^{-1}\left(\frac{1.98\times10^{20}\ \text{N}}{3.54\times10^{22}\ \text{N}}\right) = 0.321^\circ.$$

So the net force vector points almost toward the Sun. It deviates away from the Earth-Sun line only slightly.

These results show clearly that the Moon has very little effect on the motion of the Earth compared to the effect that the Sun has.

47. Conservation of energy applied to the lunar module says:

$$E_f = E_i$$

$$K_f + U_{grav}^f = K_i + U_{grav}^i$$

$$\frac{1}{2}mv_f^2 - G\frac{mM_M}{r_f} = \frac{1}{2}mv_i^2 - G\frac{mM_M}{r_i}$$

$$\frac{1}{2}mv_f^2 - G\frac{mM_M}{R_M} = \frac{1}{2}mv_i^2 - G\frac{mM_M}{R_M + h}.$$

Note: here *m* is the mass of the lunar module, M_M is the mass of the Moon, R_M is the radius of the Moon, and *h* is the initial height of the lunar module above the surface of the Moon. Solving for v_f, I get:

$$v_f = \sqrt{v_i^2 - 2GM_M\left(\frac{1}{R_M + h} - \frac{1}{R_M}\right)}$$

Plugging in numbers, I get:

$$v_f = 1.73\times10^3\ \text{m/s}$$

49. The escape speed on Earth is given by:

$$v_e = \sqrt{\frac{2GM_E}{R_E}}$$

For any other planet, we simply replace the mass and radius of the Earth with the mass and radius of the other planet. So if we call the mass of the planet *M* and the radius of the planet *R*, then:

$$v_e = \sqrt{\frac{2GM}{R}}$$

Now, we're told that the planet has a mass that's 10 times the mass of the Earth, so $M = 10M_E$. And the fact that the radius of the planet is one-tenth the radius of the Earth means that $R = \frac{R_E}{10}$. So the escape speed on this planet is:

$$v_e = \sqrt{\frac{2G\left(10M_E\right)}{\left(\frac{R_E}{10}\right)}} = \sqrt{\frac{\left(100\right)2GM_E}{R_E}} = 10\sqrt{\frac{2GM_E}{R_E}}$$

So the escape speed on this planet is 10 times the escape speed on Earth.

58. Conservation of energy says:

$$E_i = E_f$$

$$K_i + U^i_{grav} = K_f + U^f_{grav} \qquad (1)$$

Let the mass of the astronaut be *m* and the mass of the planet be *M*. Also, let the radius of the planet be *R*. Then Eq. (1) becomes:

$$\frac{1}{2}mv_i^2 - G\frac{mM}{R} = 0 - G\frac{mM}{R+h}, \qquad (2)$$

in which *h* is the height of the astronaut when she's at a *turning point*… that is, when she's momentarily *at rest* before turning around. Rearranging Eq. (2) gives:

$$v_i^2 = 2GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

And solving for *M* gives:

$$M = \frac{v_i^2}{2G\left(\frac{1}{R} - \frac{1}{R+h}\right)}$$

Plugging in numbers, I get:

$$M = \frac{\left(3.00 \text{ m/s}\right)^2}{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(\frac{1}{3.660\times10^6 \text{ m}} - \frac{1}{3.660\times10^6 \text{ m} + 0.580 \text{ m}}\right)}$$

$$M = 1.56\times10^{24} \text{ kg}$$