Physics 1401
Homework Solutions - Walker, Chapter 11

Conceptual Exercises

CE10. The puck's linear speed v increases. The angular speed ω increases. The angular momentum does not change.

Explanation: The net external force on the puck is just the tension in the string, which pulls the puck radially inward toward the center of the circle on which it moves. Since this force is *radial*, it produces *no torque* about the center. Since there is no net external torque on the puck, its angular momentum is conserved. This means that $L = I\omega$ remains fixed as the puck is pulled toward the center. As the puck is pulled closer to the center, its moment of inertia about the center goes *down*. Therefore, in order for the angular momentum to be conserved, ω must go *up*. To see that v also goes up, consider the expression for the angular momentum of the puck about the center. Treating the puck as a point mass moving with speed v in a circle of radius r, its angular momentum about the center is:

$$L = rp\sin\theta = rmv\sin 90^\circ = rmv\,.$$

Solving for v gives:

$$v = \frac{L}{rm} = \frac{\text{constant}}{r}\text{, since } L \text{ and } m \text{ are fixed.}$$

So v is found to be proportional to $\frac{1}{r}$. Therefore, as r decreases, v increases.

Problems

1. We get the *maximum* torque (for a given force) when the force is applied *tangentially* to the *end* of the wrench. In this case, the torque produced by a force F is given by:

$$\tau = Fr\,,$$

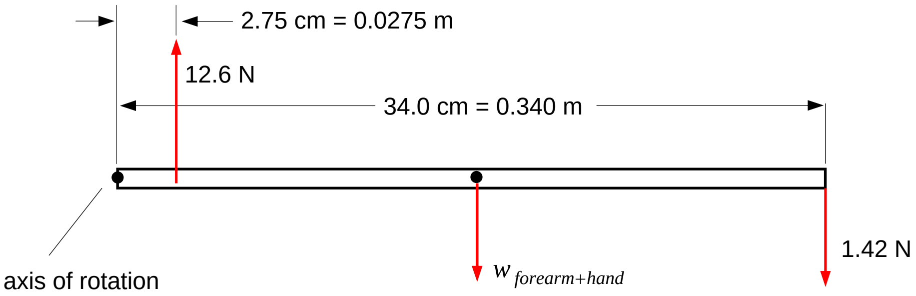
in which r is the length of the wrench. So the *minimum force* needed to produce a given *torque* is:

$$F = \frac{\tau}{r}\,.$$

So in order to produce a torque of $15\ \text{N}\cdot\text{m}$ using a wrench of length 25 cm = 0.25 m, we need a minimum force of:

$$F = \frac{15\ \text{N}\cdot\text{m}}{0.25\ \text{m}} = 60\ \text{N}\,.$$

5. (a.) Picture:



Note that the weight of the combination of forearm plus hand has been taken to act at the *center* of the rod because we are told to treat the forearm plus hand as a *uniform* rod (that is, a rod with uniform *density*).

There are three forces to consider, as shown in the picture. The net torque produced by these three forces about the axis shown is:

$$\tau_{net} = (12.6\ \mathrm{N})(0.0275\ \mathrm{m}) - w_{forearm+hand}(0.170\ \mathrm{m}) - (1.42\ \mathrm{N})(0.340\ \mathrm{m})$$

$$\tau_{net} = (12.6\ \mathrm{N})(0.0275\ \mathrm{m}) - (1.20\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(0.170\ \mathrm{m}) - (1.42\ \mathrm{N})(0.340\ \mathrm{m})$$

$$\tau_{net} = -2.14\ \mathrm{N\cdot m}\ .$$

(b.) The net torque *is* nonzero... in fact, it's *negative*. This means that the forearm and hand will rotate *clockwise*, so that the hand moves *downward*.

(c.) If the biceps exerted the same force (12.6 N) at a point farther from the elbow joint, the biceps would contribute a greater positive torque. Therefore, the net torque would *increase*.

9.

$$\tau = I\alpha$$

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.017\ \mathrm{kg})(0.060\ \mathrm{m})^2 = 3.05\times10^{-5}\ \mathrm{kg\cdot m^2}$$

Angular acceleration is *constant*. Therefore, can use any of the four facts for circular motion with constant α. In particular, we know:

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)\ .$$

Take $\theta_0 = 0$. And $\omega_0 = 0$ because the disk starts from rest. So:

$$\omega^2 = 2\alpha\theta\ ,$$

which gives:

$$\alpha = \frac{\omega^2}{2\theta}\ .$$

$$\omega = 450\ \mathrm{rev/min} = \left(\frac{450\ \mathrm{rev}}{\mathrm{min}}\right)\left(\frac{1\ \mathrm{min}}{60\ \mathrm{sec}}\right)\left(\frac{2\pi\ \mathrm{rad}}{1\ \mathrm{rev}}\right) = 47.12\ \mathrm{rad/s}\ .$$

And: $\theta = 3.0\ \mathrm{rev} = 18.85\ \mathrm{rad}$.

So:

$$\alpha = \frac{(47.12\ \mathrm{rad/s})^2}{2(18.85\ \mathrm{rad})} = 58.89\ \mathrm{rad/s^2}\ .$$

And so:

$$\tau = (3.05\times10^{-5}\ \mathrm{kg\cdot m^2})(58.89\ \mathrm{rad/s^2}) = 0.0018\ \mathrm{N\cdot m}\ .$$

13. (a.) The object experiences the greatest angular acceleration when the torque acts about the *x* axis. It experiences the least angular acceleration when the torque acts about the *z* axis. This follows from Newton's second law for rotational motion:

$$\tau_{net} = I\alpha \; .$$

Rearranging this to solve for α, I find:

$$\alpha = \frac{\tau_{net}}{I} \; .$$

So for a given net torque τ_{net}, the greatest angular acceleration α is achieved when the moment of inertia I is smallest. If we calculate the moments of inertia about the *x*, *y*, and *z* axes, we find:

$$I_x = (3.0\,\text{kg})(0.50\,\text{m})^2 + (4.0\,\text{kg})(0.50\,\text{m})^2 = 1.75\,\text{kg}\cdot\text{m}^2$$
$$I_y = (1.2\,\text{kg})(0.70\,\text{m})^2 + (4.0\,\text{kg})(0.70\,\text{m})^2 = 2.548\,\text{kg}\cdot\text{m}^2$$
$$I_z = (3.0\,\text{kg})(0.50\,\text{m})^2 + (1.2\,\text{kg})(0.70\,\text{m})^2 + (4.0\,\text{kg})\left((0.70\,\text{m})^2 + (0.50\,\text{m})^2\right) = 4.298\,\text{kg}\cdot\text{m}^2$$

The smallest moment of inertia is the moment of inertia about the *x* axis. Therefore, the object will have the greatest angular acceleration when it is rotated about this axis. The largest moment of inertia is the moment of inertia about the *z* axis. Therefore, the object will have the least angular acceleration when it is rotated about this axis.

(b.) If the torque is $13\,\text{N}\cdot\text{m}$ about the *x* axis, the angular acceleration about the *x* axis will be:

$$\alpha_x = \frac{\tau}{I_x} = \frac{13\,\text{N}\cdot\text{m}}{1.75\,\text{kg}\cdot\text{m}^2} = 7.4\,\text{rad/s}^2 \; .$$

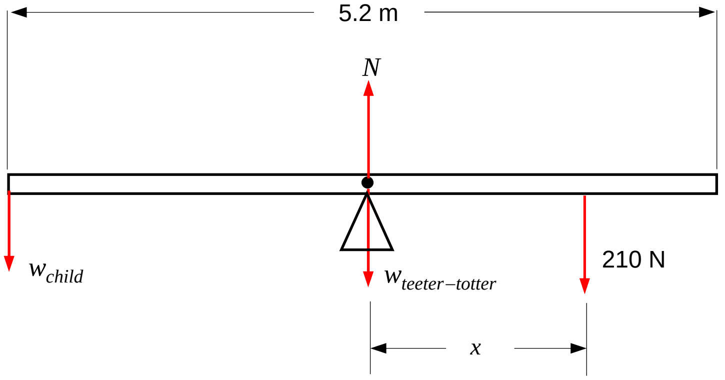
(c.) For the same torque, but applied instead about the *y* axis:

$$\alpha_y = \frac{\tau}{I_y} = \frac{13\,\text{N}\cdot\text{m}}{2.548\,\text{kg}\cdot\text{m}^2} = 5.1\,\text{rad/s}^2 \; .$$

(d.) For the same torque, but applied instead about the *z* axis:

$$\alpha_z = \frac{\tau}{I_z} = \frac{13\,\text{N}\cdot\text{m}}{4.298\,\text{kg}\cdot\text{m}^2} = 3.0\,\text{rad/s}^2 \; .$$

22. Picture:



(a.) Net torque about an axis through the center of the teeter-totter is:

$$\tau_{net} = w_{child}(2.6\text{ m}) - (210\text{ N})x = (18\text{ kg})(9.81\text{ m/s}^2)(2.6\text{ m}) - (210\text{ N})x\,.$$

And we want this to equal zero. So:

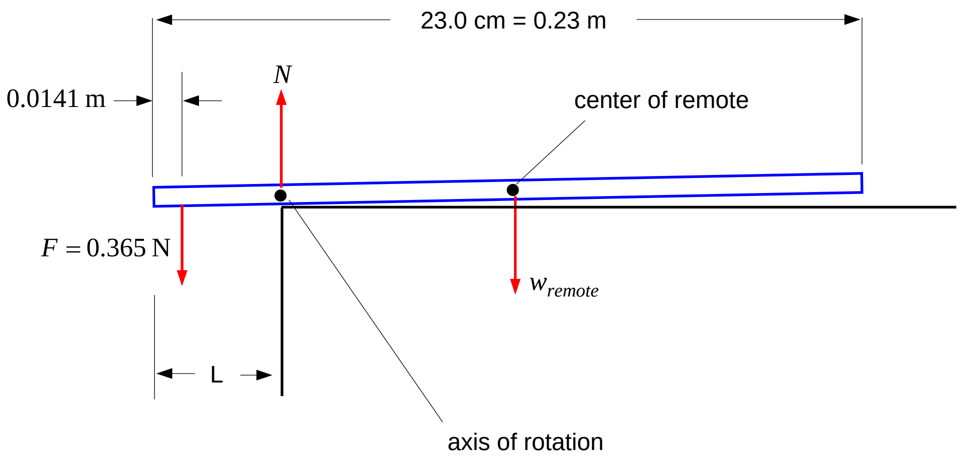
$$(18\text{ kg})(9.81\text{ m/s}^2)(2.6\text{ m}) - (210\text{ N})x = 0$$
$$x = \frac{(18\text{ kg})(9.81\text{ m/s}^2)(2.6\text{ m})}{210\text{ N}} = 2.2\text{ m.}$$

(b.) If parent pushes with a force of 310 N, then:

$$x = \frac{(18\text{ kg})(9.81\text{ m/s}^2)(2.6\text{ m})}{310\text{ N}} = 1.5\text{ m.}$$

(c.) The only thing that changing the mass of the teeter-totter would do is change the *weight* of the teeter-totter. But this has absolutely no effect on the torque about the *center* of the teeter-totter. (The weight of the teeter-totter never produces any torque about the *center* of the teeter-totter because the weight acts directly *at* the center of the teeter-totter.... i.e., the *moment arm* for the weight is *zero*.) So there would be no change in the expression for the net torque, and therefore no change in the answers we found.

23. When the remote has *just begun* to tip, the picture looks like this:



Notice that I've chosen the axis of rotation to be through the point about which the remote pivots. By choosing the axis of rotation to be this axis, I free myself from having to calculate the normal force N, since N *doesn't produce any torque* about this axis. So the expression for the net torque about this axis is:

$$\tau_{net} = (0.365\text{ N})(L - 0.0141\text{ m}) - w_{remote}(0.115\text{ m} - L)$$
$$\tau_{net} = (0.365\text{ N})(L - 0.0141\text{ m}) - (0.122\text{ kg})(9.81\text{ m/s}^2)(0.115\text{ m} - L).$$

If the remote is not to tip over, then this net torque must equal zero:

$$(0.365\text{ N})(L - 0.0141\text{ m}) - (0.122\text{ kg})(9.81\text{ m/s}^2)(0.115\text{ m} - L) = 0.$$

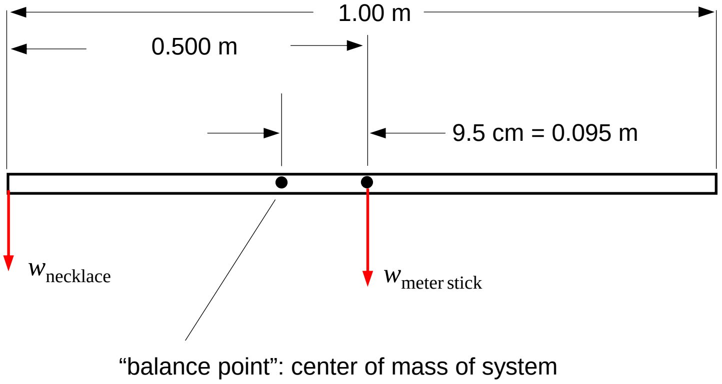
Thus the maximum L in order for the remote not to tip over is found by solving the equation immediately above for L:

$$L\left[0.365\text{ N} + (0.122\text{ kg})(9.81\text{ m/s}^2)\right] - (0.365\text{ N})(0.0141\text{ m}) - (0.122\text{ kg})(9.81\text{ m/s}^2)(0.115\text{ m}) = 0$$

$$L = \frac{(0.365\text{ N})(0.0141\text{ m}) + (0.122\text{ kg})(9.81\text{ m/s}^2)(0.115\text{ m})}{\left[(0.365\text{ N}) + (0.122\text{ kg})(9.81\text{ m/s}^2)\right]} = 0.0914\text{ m} = 9.14\text{ cm}.$$

38. If the meter stick balances at its *center*, then the *center of mass* of the meter stick is at the center, which means that the mass is distributed *uniformly* throughout the meter stick. This means that we can take the weight of the meter stick to be concentrated at the center.

Now consider the situation when the necklace is placed at one end of the meter stick. Once the necklace is placed on the meter stick, we are told that the balance point (the center of mass of the *system* of meter stick plus necklace) is located 9.5 cm away from the "old" center of mass – namely the center of the meter stick – in a direction toward the end at which the necklace hangs. So the picture looks like this:



Now suppose we support the meter stick (with the necklace hanging from it) by putting a finger (or some other support) directly underneath the center of mass of the system. Because this is the "balance point" of the system, the net torque about this point will be zero:

$$\tau_{net} = 0$$

$$w_{\text{necklace}}\left(0.500\text{ m} - 0.095\text{ m}\right) - w_{\text{meter stick}}\left(0.095\text{ m}\right) = 0$$

$$w_{\text{necklace}} = w_{\text{meter stick}}\frac{\left(0.095\text{ m}\right)}{\left(0.500\text{ m} - 0.095\text{ m}\right)} = w_{\text{meter stick}}\frac{\left(0.095\text{ m}\right)}{\left(0.405\text{ m}\right)} \text{ .}$$

So:

$$m_{\text{necklace}}\cancel{g} = m_{\text{meter stick}}\cancel{g}\frac{\left(0.095\text{ m}\right)}{\left(0.405\text{ m}\right)}$$

$$m_{\text{necklace}} = \left(0.34\text{ kg}\right)\frac{\left(0.095\text{ m}\right)}{\left(0.405\text{ m}\right)} = 0.080\text{ kg} = 80\text{ g .}$$

56. The total angular momentum of the system about an axis through the center of the hoop is:

$$L_{total} = L_{hoop} + L_{gerbils} \text{ .}$$

But the gerbils don't contribute to the angular momentum about this axis because they aren't *moving* relative to this axis. (They're *running in place*.) So the total angular momentum of the system is just the angular momentum of the hoop about an axis through its center:

$$L_{total} = L_{hoop} = I_{hoop}\omega = \left(M_{hoop}R_{hoop}^2\right)\omega_{hoop} \text{ .}$$

We know $M_{hoop} = 5.0\text{ g} = 0.0050\text{ kg}$ and $R_{hoop} = 9.5\text{ cm} = 0.095\text{ m}$, but we need ω_{hoop}. Well, that's easy: ω_{hoop} is related to the tangential speed of each gerbil (the number of meters per second covered by each gerbil) by the relationship:

$$v_{gerbil} = R_{hoop}\omega_{hoop} \ .$$

So:
$$\omega_{hoop} = \frac{v_{gerbil}}{R_{hoop}} ,$$

and:
$$L_{total} = \left(M_{hoop}R_{hoop}^2\right)\left(\frac{v_{gerbil}}{R_{hoop}}\right) = M_{hoop}R_{hoop}v_{gerbil}$$

$$L_{total} = (0.0050\ \text{kg})(0.095\ \text{m})(0.55\ \text{m/s}) = 2.6\times10^{-4}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}} \ .$$

65. (a.) Faster, because the moment of inertia of the system will have decreased, but $L = I\omega$ must stay the same, since the mouse's walking to the center does not exert a net external torque on the system.

(b.)
$$L_i = \left(I_{table} + I_{mouse}^i\right)\omega_i = \left(I_{table} + m_{mouse}R_{table}^2\right)\omega_i$$
$$L_i = \left(5.4\times10^{-3}\ \text{kg}\cdot\text{m}^2 + (0.036\ \text{kg})(0.15\ \text{m})^2\right)(33.33333\ \text{rpm})$$

And:
$$L_f = I_{table}\omega_f = \left(5.4\times10^{-3}\ \text{kg}\cdot\text{m}^2\right)\omega_f$$

(mouse is at the *center* of the table ... $r = 0$ ... so he doesn't contribute anything to the final angular momentum)

Since the angular momentum is conserved, we have:

$$\left(5.4\times10^{-3}\ \text{kg}\cdot\text{m}^2\right)\omega_f = \left(5.4\times10^{-3}\ \text{kg}\cdot\text{m}^2 + (0.036\ \text{kg})(0.15\ \text{m})^2\right)(33.33333\ \text{rpm}) \ ,$$

which gives:
$$\omega_f = 38.33333\ \text{rpm} \ .$$

Converting to rad/s, I find:

$$\left(\frac{38.33333\ \text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 4.0\ \text{rad/s} \ .$$