

Conceptual Exercises

CE4. (a.) The ratio of Jason's angular speed to Betsy's angular speed is 1. (They have the *same* angular speed because all parts of the merry-go-round must go through the same number of radians per second. Otherwise, the merry-go-round would not behave as a *rigid* object.)

$$\begin{aligned} \text{(b.) } v_{Jason} &= r_{Jason} \omega = R\omega \\ v_{Betsy} &= r_{Betsy} \omega = (2R)\omega \end{aligned}$$

$$\text{So: } \frac{v_{Jason}}{v_{Betsy}} = \frac{R\omega}{2R\omega} = \frac{1}{2} .$$

$$\begin{aligned} \text{(c.) } (a_{cp})_{Jason} &= \frac{v_{Jason}^2}{r_{Jason}} = r_{Jason} \omega^2 = R\omega^2 \\ (a_{cp})_{Betsy} &= r_{Betsy} \omega^2 = 2R\omega^2 \end{aligned}$$

$$\text{So: } \frac{(a_{cp})_{Jason}}{(a_{cp})_{Betsy}} = \frac{R\omega^2}{2R\omega^2} = \frac{1}{2} .$$

Problems

$$10. \text{ (a.) } T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{2.00 \times 10^{-1} \text{ s}} = 31.4 \text{ rad/s} .$$

(b.) They tell us that a "3.5-inch" floppy disk is 3.5 inches in *diameter*. Therefore, the disk's *radius* is one-half this:  $R_{disk} = 1.75 \text{ in}$  . So the linear speed of a point on the rim of the disk is:

$$v = R_{disk} \omega = (1.75 \text{ in})(31.4 \text{ rad/s}) = 54.95 \text{ in/s} .$$

The book probably wants the speed in *meters* per second, not *inches* per second, so I'll do a unit conversion:

$$\left( \frac{54.95 \text{ in}}{\text{s}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) = 1.40 \text{ m/s} .$$

But I should really have kept only two significant figures in  $R_{disk} = 1.75 \text{ in}$  , since 3.5 in has just two significant figures. So to two significant figures, the answer I get for  $v$  is  $v = 1.4 \text{ m/s}$  .

(c.) The same. All parts of the disk have to go through the same *angle* per unit time if the disk is to behave *rigidly* (i.e., if it's to maintain its *shape*). So if one part of the disk goes through an angle of 31.4 rad in one second, then *all* parts of the disk must go through an angle of 31.4 rad in the same amount of time... Otherwise, points in one part of the disk are "getting ahead" of points in another part of the disk.

20. (a.) Constant  $\alpha$  . One of the "four facts" says:

$$\omega = \omega_0 + \alpha t .$$

Solve this for  $\alpha$  :

$$\alpha = \frac{\omega - \omega_0}{t} .$$

If  $t$  is the time when the centrifuge *stops*, then  $\omega = 0$  . So:

$$\alpha = \frac{-\omega_0}{t} = \frac{-3850 \text{ rpm}}{10.2 \text{ s}} .$$

At this point, we have a problem with *units*: I want  $\alpha$  in  $\text{rad/s}^2$  , but I've got *revolutions* instead of *radians* and *minutes* (in the numerator) instead of *seconds*. Well, no problem. One *revolution* is equal to  $2\pi$  *radians*. And we know how to convert from minutes to seconds. So here goes the conversion of 3850 rpm to rad/s:

$$\left( \frac{3850 \text{ rev}}{\text{min}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 403.171 \text{ rad/s} .$$

OK. So:

$$\alpha = \frac{-403.171 \text{ rad/s}}{10.2 \text{ s}} = -39.5 \text{ rad/s}^2 \quad (\text{keeping three sig figs}).$$

So the magnitude of the angular acceleration is  $39.5 \text{ rad/s}^2$  .

- (b.) This is like saying, "Find the angle  $\theta$  when  $\omega = 0$  ," except that they want the angle in *revolutions*, not *radians*. Well, I know:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 .$$

Taking  $\theta_0$  to be zero, I have:

$$\begin{aligned} \theta &= \omega_0 t + \frac{1}{2} \alpha t^2 = (403.171 \text{ rad/s})(10.2 \text{ s}) + \frac{1}{2} (-39.5 \text{ rad/s}^2)(10.2 \text{ s})^2 \\ \theta &= 2058 \text{ rad} . \end{aligned}$$

Now convert this to revolutions:

$$(2058 \text{ rad}) \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 327 \text{ rev} \quad (\text{keeping 3 sig figs.})$$

26. (a.) If the angular speed is *constant*, then  $\omega = \omega_{av} = \frac{\Delta\theta}{\Delta t}$  . (They want us to *assume*  $\omega$  is constant, evidently, since they don't tell us the rate at which  $\omega$  is changing... i.e., they don't tell us the angular *acceleration*.) So we'll play along and assume that  $\omega$  is constant. Then:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{1 \text{ rev}}{4.5 \text{ s}} = 0.222 \text{ rev/s} .$$

This is the angular speed in *revolutions* per second. If we wanted it in *radians* per second, then we'd convert:

$$\left( \frac{0.222 \text{ rev}}{\text{s}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 1.4 \text{ rad/s} .$$

The angular speed is the *same* for the two children because the system of the children plus the merry-go-round behaves as a *rigid* object.

$$(b.) \quad v_{\text{Child 1}} = r_{\text{Child 1}}\omega = (2.0 \text{ m})(1.4 \text{ rad/s}) = 2.8 \text{ m/s} .$$

$$v_{\text{Child 2}} = r_{\text{Child 2}}\omega = (1.5 \text{ m})(1.4 \text{ rad/s}) = 2.1 \text{ m/s} .$$

41. The phrase “the car is driven at 12 m/s” implies that, at each instant, the *center* of each tire is moving forward with linear speed  $v = 12 \text{ m/s}$  . But we know that for rolling without slipping, this translational speed of the center of the wheel is related to the angular speed of the entire wheel by:

$$v = R\omega .$$

So the angular speed of the tires is:

$$\omega = \frac{v}{R} = \frac{12 \text{ m/s}}{31 \text{ cm}} ,$$

except that now we’ve got a problem... We’ve got two different units being used for length (meters and centimeters). So we need to get a consistent set of units. So express 31 cm as 0.31 m, and the problem is solved:

$$\omega = \frac{v}{R} = \frac{12 \text{ m/s}}{0.31 \text{ m}} = 39 \text{ rad/s} .$$

(Notice that the units you’d get by carrying out the division indicated immediately above would be  $\frac{1}{\text{s}}$  . But remember that a radian is really a dimensionless unit. So what we do is simply *supply* the units “rad” where they are needed in the expression above in order that  $\omega$  be expressed in rad/s. This does not alter the result in any way.)

50. (a.)  $K_{\text{rot}} = \frac{1}{2}I\omega^2$

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.013 \text{ kg})(0.060 \text{ m})^2 = 2.34 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

$$\text{So: } K_{\text{rot}} = \frac{1}{2}(2.34 \times 10^{-5} \text{ kg} \cdot \text{m}^2)(32 \text{ rad/s})^2 = 0.012 \text{ J} .$$

- (b.) Because  $K_{\text{rot}}$  is proportional to the *square* of  $\omega$  , if  $K_{\text{rot}}$  is to be doubled,  $\omega$  must be increased by a factor of  $\sqrt{2}$  . So we need:

$$\omega = \sqrt{2}(32 \text{ rad/s}) = 45 \text{ rad/s} .$$

63. (a.) The total energy is conserved. Choosing the zero of the gravitational potential energy to be at the bottom of the ramp gives:

$$E_i = Mgh$$

and

$$E_f = K_{trans} + K_{rot} = K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2.$$

The total initial energy is:

$$E_i = (2.0 \text{ kg})(9.81 \text{ m/s}^2)(0.75 \text{ m}) = 14.715 \text{ J}.$$

So since the energy is conserved, this must equal the total *kinetic* energy at the *bottom* of the ramp:

$$E_f = E_i$$

$$K_{total} = E_i = 14.715 \text{ J} = 15 \text{ J}, \text{ keeping 2 sig figs.}$$

- (b.) Since the cylinder rolls without slipping, the translational velocity  $v$  is related to the angular velocity  $\omega$  by:

$$v = R\omega.$$

or:

$$\omega = \frac{v}{R}.$$

Therefore the rotational kinetic energy (about the center of mass) is:

$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \left(\frac{1}{2}\right)\left(\frac{1}{2}Mv^2\right).$$

So the total kinetic energy is:

$$K_{total} = K_{trans} + K_{rot} = \frac{1}{2}Mv^2 + \left(\frac{1}{2}\right)\left(\frac{1}{2}Mv^2\right) = \left(\frac{3}{2}\right)\left(\frac{1}{2}Mv^2\right) = \frac{3}{2}K_{trans}.$$

But we know  $K_{total} = 14.715 \text{ J}$ , so:

$$K_{trans} = \frac{2}{3}(14.715 \text{ J}) = 9.8 \text{ J}, \text{ keeping 2 sig figs.}$$

So the *rotational* kinetic energy is:

$$K_{rot} = K_{total} - K_{trans} = K_{total} - \frac{2}{3}K_{total} = \frac{1}{3}K_{total} = \frac{1}{3}(14.715 \text{ J}) = 4.9 \text{ J}$$

- (c.) I just calculated this in Part (b.).

68.

$$K_{total} = K_{trans} + K_{rot} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

But  $I = MR^2$  (moment of inertia of hoop about center of mass) and  $\omega = \frac{v}{R}$  (since hoop rolls without slipping),  
so:

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(MR^2\right)\left(\frac{v}{R}\right)^2$$

$$K_{total} = Mv^2 .$$

So if  $K_{total}$  is to equal 0.10 J,

$$Mv^2 = 0.10 \text{ J}$$

So:

$$v = \sqrt{\frac{0.10 \text{ J}}{0.050 \text{ kg}}} = 1.4 \text{ m/s} .$$

95. (a) To find the distance  $d$ , I would like to find the translational speed of the ball at the instant it reaches the bottom of the ramp and leaves the table. From this point on, the ball is just a projectile, so I can use all of the facts I know about projectile motion. So let's find the speed of the ball at the bottom of the ramp. To do so, I'll use energy conservation. Since the ball starts from rest at the top of the ramp, its total initial energy is just its gravitational potential energy. Taking the zero of the potential energy to be at the bottom of the ramp, the initial energy of the ball is:

$$E_i = Mgh = Mg(0.61 \text{ m}) .$$

When the ball reaches the bottom of the ramp, its energy is all *kinetic*. But it has two kinds of kinetic energy – translational and rotational. So the total energy at the bottom of the ramp is:

$$E_f = K_{trans} + K_{rot} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 ,$$

in which  $\frac{1}{2}Mv^2$  is the *translational* kinetic energy of the center of mass and  $\frac{1}{2}I\omega^2$  is the *rotational* kinetic energy *about* the center of mass. The moment of inertia of a solid sphere about an axis through its center of mass is, from Table 10-1,

$$I = \frac{2}{5}MR^2 .$$

So:

$$E_f = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 ,$$

in which I've used the relation  $v = \omega R$ , or  $\omega = \frac{v}{R}$ . Simplifying the expression immediately above gives:

$$E_f = \frac{7}{10}Mv^2 .$$

And this must be equal to  $E_i$  (by conservation of energy), so:

$$\frac{7}{10}Mv^2 = Mg(0.61 \text{ m}),$$

from which I find that:

$$v = \sqrt{\frac{10}{7}(9.81 \text{ m/s}^2)(0.61 \text{ m})} = 2.924 \text{ m/s}.$$

This is the speed of the ball when it leaves the table and becomes a projectile.

Now let's consider this second part of the motion (the projectile motion part). Let the origin of an  $x$ - $y$  coordinate system be chosen to be where the ball is at the instant it leaves the table. Also, let's let the *time* at this instant be  $t = 0$ . Then from what we know about the motion of projectiles, the horizontal position at any time  $t$  will be given by:

$$x = x_0 + v_{x0}t$$

or:

$$x = (2.924 \text{ m/s})t,$$

since  $x_0 = 0$  (by the choice of origin of the coordinate system) and because the velocity is completely in the horizontal direction at  $t = 0$ .

So we need to find  $t$  when ball hits the floor. To do this, as always, consider the  $y$  motion. We know that the position in the  $y$  direction is given by:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2,$$

and we would like to know, "What is  $t$  when  $y = -1.22 \text{ m}$ ?" Well, the answer, from the expression immediately above, is:

$$t = \sqrt{-\frac{2y}{g}} = \sqrt{-\frac{2(-1.22 \text{ m})}{9.81 \text{ m/s}^2}} = 0.4987 \text{ s}.$$

So the distance  $d$  is:

$$d = (2.924 \text{ m/s})(0.4987 \text{ s}) = 1.5 \text{ m},$$

keeping 2 sig figs.

- (b.) While the ball is falling, its angular velocity *does not change* (there's no *torque* on it while it's falling), so the ball's angular velocity while it's falling will be whatever it was at the instant the ball reached the bottom of the ramp. So let's find  $\omega$  at the bottom of the ramp. Well, I figure  $\omega$  is related to  $v$  by

$\omega = \frac{v}{R}$ , and I've already found that  $v$  at the bottom of the ramp is 2.924 m/s, so  $\omega$  at the bottom of the ramp must be:

$$\omega = \frac{2.924 \text{ m/s}}{0.085 \text{ m}} = 34.4 \text{ rad/s},$$

Physics 1401  
Homework Solutions - Walker, Chapter 10

in which I've found the radius of the ball by dividing the diameter (given in the problem as 0.17 m) by two.

So the ball's angular velocity is equal to 34.4 rad/s the entire time it is falling. We know that the time it takes to fall is 0.4987 s. So what angle does it go through in this time? Well,

$$\omega = \frac{\Delta\theta}{\Delta t},$$

so: 
$$\Delta\theta = \omega \Delta t = (34.4 \text{ rad/s})(0.4987 \text{ s}) = 17.2 \text{ rad}.$$

This angle corresponds to a number of revolutions given by:

$$\Delta\theta(\text{rev}) = (17.2 \text{ rad}) \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 2.7 \text{ rev},$$

keeping 2 sig figs.

- (c.) If the ramp were to be made frictionless, the distance  $d$  would *increase*. If the ramp were frictionless, the sphere would not spin, it would just *slide*. This means that the rotational kinetic energy would be zero. This would mean that more – in fact, *all* – of the sphere's initial energy (the gravitational potential energy it has at the top of the ramp) would be converted into *translational* kinetic energy at the bottom of the ramp, and no energy would have to go into *rotational* kinetic energy. So there would be more energy available for translation, and therefore the sphere would have a greater speed at the bottom of the ramp, and it would go a greater horizontal distance  $d$ .