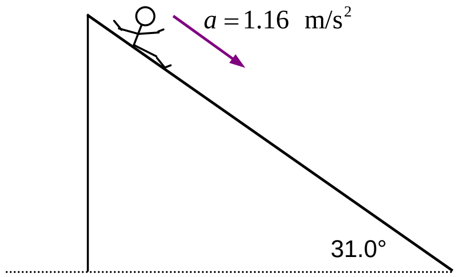
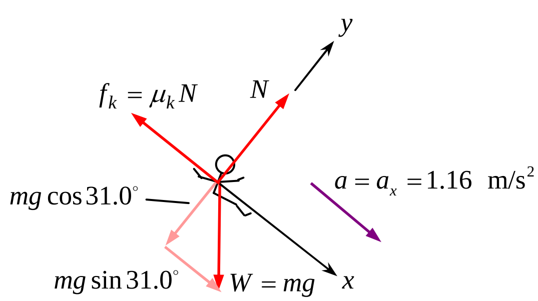
Physics 1401
Homework Solutions - Walker, Chapter 6

Conceptual Questions
CQ15. Sitting on the trunk of the car helps because it increases the normal force that the ground exerts on each patch of tire that is in contact with the ground. This increases the force of static friction, since $f_s$ is proportional to the normal force ( $f_s \le \mu_s N$ ).

Problems
2. Picture:




Consider FBD for the child:




Newton's second law applied to the child gives:

$$\sum F_x = ma_x$$

$$\sum F_y = 0$$

Consider the $x$ direction. Filling in the forces in the $x$ direction, I get:

$$mg\sin 31.0^\circ - \mu_k N = m\left(1.16 \text{ m/s}^2\right)$$

Solving for $\mu_k$, I get:
$$\mu_k = \frac{mg\sin 31.0^\circ - m\left(1.16 \text{ m/s}^2\right)}{N}.$$

But I need to know what $N$ is. I get it from the $y$ direction:

$$\sum F_y = 0$$

$$N - mg\cos 31.0^\circ = 0$$

$$N = mg\cos 31.0^\circ \ .$$

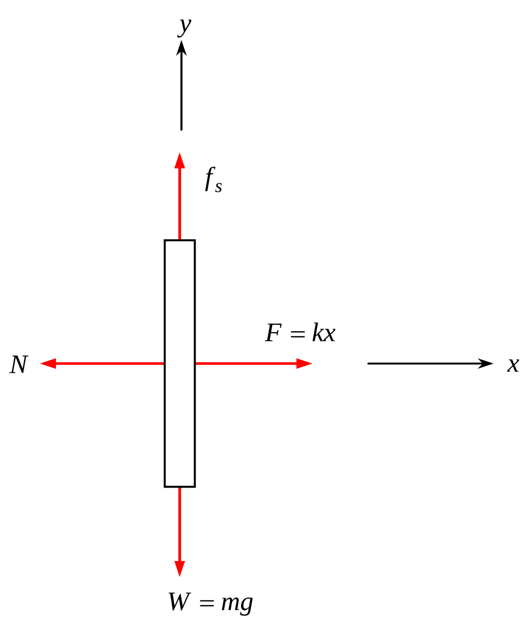
Put this into the expression for $\mu_k$ :

$$\mu_k = \frac{mg\sin 31.0^\circ - m\left(1.16 \ \text{m/s}^2\right)}{mg\cos 31.0^\circ} = \frac{mg\sin 31.0^\circ}{mg\cos 31.0^\circ} - \frac{m\left(1.16 \ \text{m/s}^2\right)}{mg\cos 31.0^\circ}$$

$$\mu_k = \tan 31.0^\circ - \frac{1.16 \ \text{m/s}^2}{\left(9.81 \ \text{m/s}^2\right)\cos 31.0^\circ}$$

$$\mu_k = 0.463$$

23. Hooke's law says that in order to compress the spring a distance $x$ and hold it there, the person's hand will have to exert a force of magnitude $kx$. By Newton's third law, the spring will push back on the person's hand with a force of equal magnitude, $kx$. But – and here is the key point, as far as the block is concerned – the spring will *also* push *on the block* with a force of magnitude $kx$. (That is, when it is compressed a distance $x$, the spring pushes on the objects attached to *either of its ends* with a force of magnitude $kx$.) So the free-body diagram for the block will look like:




(a.) The problem asks for the *minimum* compression needed to keep the block from falling. Suppose we start by compressing the spring quite a lot (more than is necessary to keep the block from falling), and then gradually let the spring expand so that it is not compressed quite as much. As we let the spring expand toward its relaxed length, the force it exerts on the block will decrease. At some point, the block will be *on the verge* of falling. At this point, we have the *minimum* compression necessary to keep the block from falling – any *less* compression, and it will fall. What is the condition for the block's being *on the verge* of falling? The condition is that $f_s = \mu_s N$ .

Applying Newton's second law to the block in the *x* and *y* directions, I find:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

Let's consider the *x* direction first:

$$kx - N = 0$$

Which gives: $$x = \frac{N}{k} .$$

But what is *N*? Well, consider the *y* direction:

$$\mu_s N - mg = 0$$

$$N = \frac{mg}{\mu_s} .$$

Plugging this into the expression for *x* gives:

$$x = \frac{mg}{\mu_s k} \qquad (1)$$

So the minimum compression needed is:

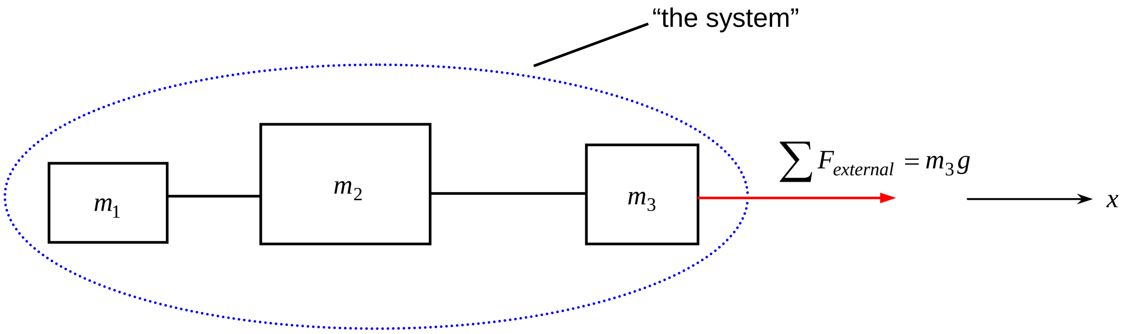
$$x = \frac{(0.27 \text{ kg})(9.81 \text{ m/s}^2)}{(0.46)(120 \text{ N/m})} = 0.048 \text{ m} = 4.8 \text{ cm}$$

(b.) Yes. *x* is directly proportional to the mass, so if the mass were doubled, the minimum compression would be doubled. (See Equation 1 above.) To put it another way, if the mass were doubled, $f_s$ would have to go up by a factor of two (to keep the block in equilibrium in the vertical direction). But that means *N* would have to go up by a factor of two. But *that* means that *kx* would have to go up by a factor of two. So *x* would have to go up by a factor of two.

36. This problem is almost *trivial*, if you think about the *entire system* composed of $m_1$, $m_2$, $m_3$, and the two strings (and this *is* how they *intend* for you to think about the problem). As far as this system is concerned, the net *external* force is just the gravitational force that the earth exerts on $m_3$ ... that is, the *weight* of $m_3$. (The tensions in the strings are *internal* to this system.) Now, Newton's second law says:

$$\sum \vec{F} = m\vec{a} ,$$

that is, the net force on an object (or a *system* of objects) equals the mass of the object (or system) times the acceleration of the object (or system). In addition, the net force that appears in Newton's second law is the net *external* force on the object (or system). So if we have a system of objects, only the *external* forces lead to acceleration of the whole system. (This point was never worth making when we were talking about forces on a *single object* because in that case the "system" consisted of just one object, so *every* force on that object was an external force.) So for the three masses in this problem, it's *as though* you had the following situation:




So what's the acceleration of this system? Well, Newton's second law applied to the system says (for the $x$ direction):

$$m_3 g = (m_1 + m_2 + m_3) a_{system} .$$

So:

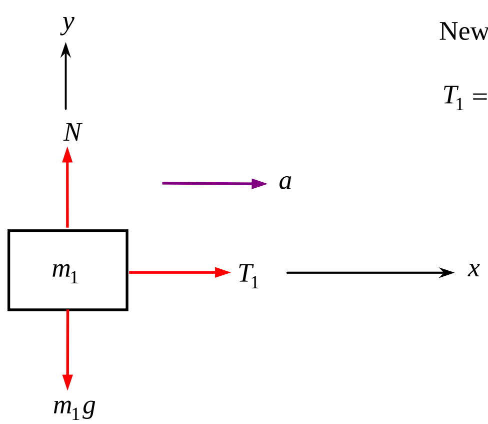
$$a_{system} = \frac{m_3 g}{(m_1 + m_2 + m_3)} = \frac{3}{6}\left(9.81\,\text{m/s}^2\right), \text{ or}$$

$$a_{system} = 4.91\,\text{m/s}^2 .$$

Easy, right?

As a check, we might work this problem in a more familiar way. Draw FBDs for each individual mass. Let the tension in the left string be called $T_1$ and the tension in the right string be called $T_2$. Then the FBDs look like:

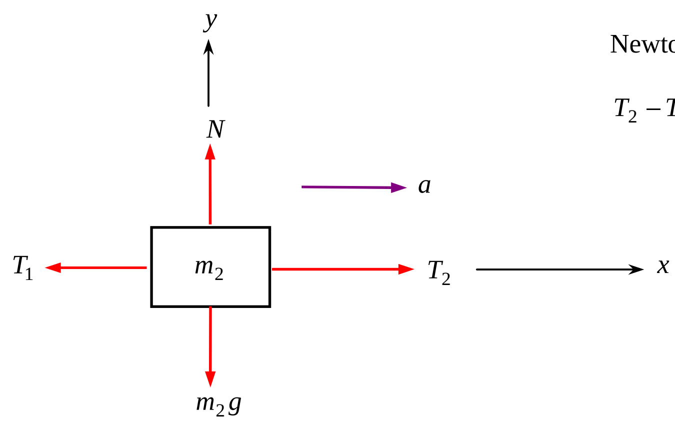
For $m_1$:




Newton's second law says:

$T_1 = m_1 a$ (1)

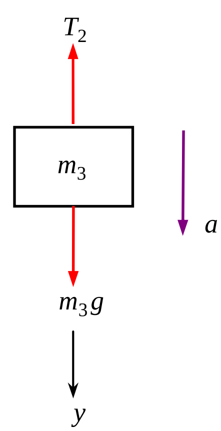
For $m_2$:




Newton's second law says:

$$T_2 - T_1 = m_2 a \qquad (2)$$

And for $m_3$:




Newton's second law says:

$$m_3 g - T_2 = m_3 a \qquad (3)$$

Now do a little algebra to solve for *a*: Substitute (1) into (2). Then (2) becomes:

$$T_2 - m_1 a = m_2 a \text{ , or}$$

$$T_2 = (m_1 + m_2)a \text{ .}$$

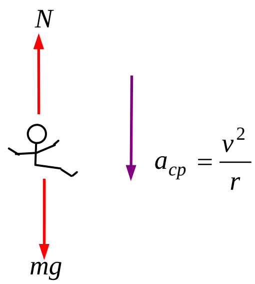
Then substitute this into (3) to get:

$$m_3 g - (m_1 + m_2)a = m_3 a \text{ , or:}$$

$$a = \frac{m_3 g}{(m_1 + m_2 + m_3)},$$

which is exactly the same result we got by the first method!

49. (a.) At the *top* of the Ferris wheel, your FBD looks like:




So Newton's second law says: $mg - N = m\frac{v^2}{r}$, which gives the apparent weight (the normal force) as:

$$N = mg - m\frac{v^2}{r} \ .$$

At the *bottom* of the Ferris wheel, the FBD looks the same, as far as the forces on you are concerned, but the centripetal acceleration is now *upward* (the center of the Ferris wheel is now *above* you), so Newton's second law gives:

$$N - mg = m\frac{v^2}{r} \text{, or}$$

$$N = mg + m\frac{v^2}{r}$$

So your apparent weight at the bottom is greater than what it is at the top. This is why you feel "heavier" at the bottom and "lighter" at the top.

(b.) This is just plugging in numbers, except for one thing: What is $v$? Well, if the Ferris wheel completes one revolution every 28 s, then the *distance* it goes in 28 s is the *circumference* of the Ferris wheel, $2\pi r$. So the *speed* of the Ferris wheel (assuming this speed is constant, so we can just use the definition of *average* speed) is:

$$v = \frac{\text{total distance}}{\text{total time}} = \frac{2\pi(7.2\text{ m})}{28\text{ s}} = 1.616\text{ m/s} \ .$$

So the apparent weight at the top is:

$$N = mg - m\frac{v^2}{r} = (55\text{ kg})(9.81\text{ m/s}^2) - (55\text{ kg})\frac{(1.616\text{ m/s})^2}{7.2\text{ m}} = 520\text{ N} \ .$$

At the bottom, the apparent weight is:

$$N = mg + m\frac{v^2}{r} = 559\text{ N} \ .$$

51. Referring to Problem 50, I find that it bears a striking similarity to the situation we just discussed: It's just like going over the top of the Ferris wheel. So we'll have (by the same arguments I just made for Problem 49):

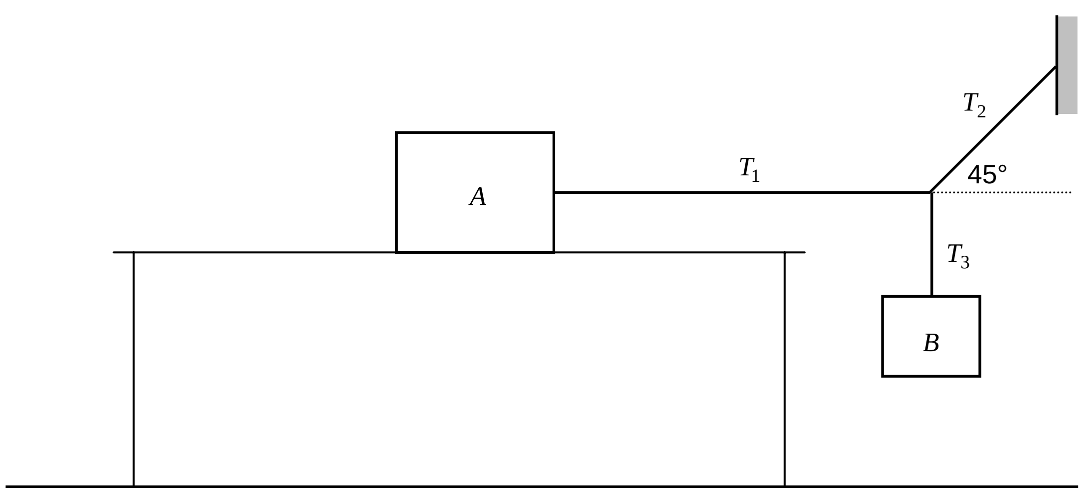
$$N = mg - m\frac{v^2}{r},$$

and we'd like to know what $v$ needs to be in order for the people in the car to *feel* "weightless." (I want to put emphasis on the word "feel" here.... the people in the car are *never really* weightless, of course.... they *always* have weight equal to their mass times $g$.) Well, the key to this problem is to realize that the people *feel* weightless when they no longer feel the seat pushing upward on them.... in other words, when $N$ becomes zero. You can think of it this way, if you like: If you were to go over the bump repeatedly, each time increasing your speed, the term $m\frac{v^2}{r}$ in the above expression for $N$ would be larger each time. At some point, you'd go over the bump with a speed that would make the term $m\frac{v^2}{r}$ equal to $mg$. As you go over the top of the bump at this speed, the normal force goes to zero, according to the expression for $N$ above. In this condition, the people in the car no longer feel the seat pushing upward on them. The people actually *leave the surface of the seat* .... they go into *free-fall* for a short time, until they fall back down to the seat. The speed required for this to happen is then given by:

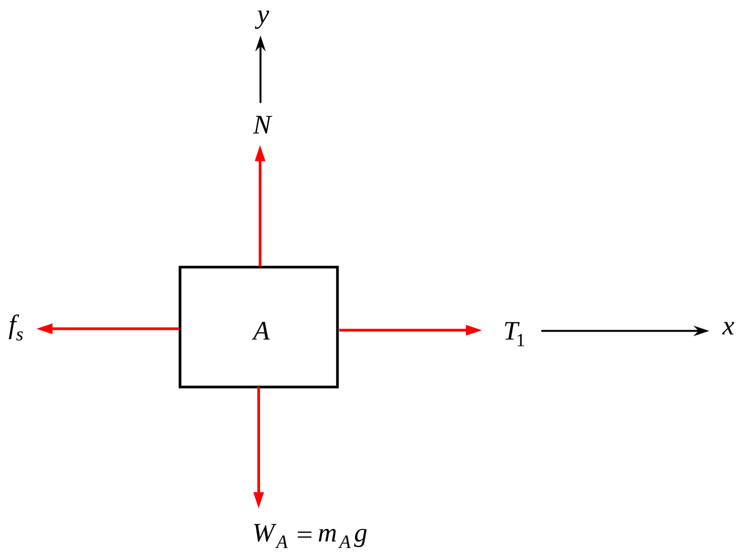
$$m\frac{v^2}{r} = mg \text{ , or:}$$

$$v = \sqrt{gr} = \sqrt{(9.81)(35)} = 19 \text{ m/s}$$

67. A diagram of the situation is shown below. Let the tensions in the three ropes be called $T_1$, $T_2$, and $T_3$, as indicated in the diagram.




Free-body diagram for block A:



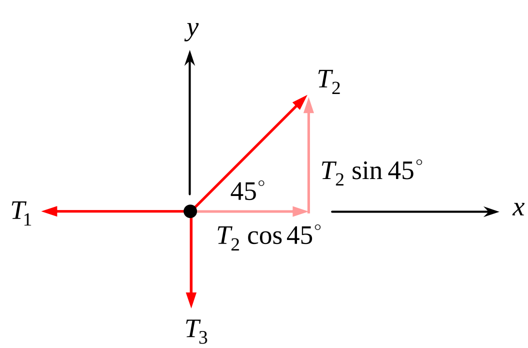

(a.) We want to find $f_s$. Well, applying Newton's second law to Block A in the *x* direction, I find:

$$\sum F_x = 0$$

$$T_1 - f_s = 0$$

$$f_s = T_1.$$

So we need to find $T_1$. (Notice that we can't say $f_s = \mu_s N$ here because we are not told that Block A is *on the verge* of sliding.) Now, how to get $T_1$? Well, there is another object that $T_1$ acts on: the *knot* where the three strings come together. This knot is in equilibrium, so the sum of all the forces on it must be zero. Consider the FBD for the knot:



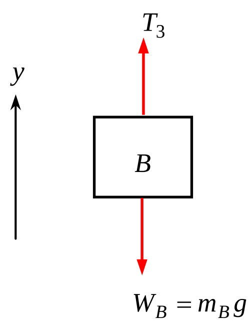

Applying Newton's second law to the knot in the *x* direction, I find:

$$T_1 = T_2 \cos 45^\circ.$$

But what is $T_2$? Well, consider the forces on the knot in the *y* direction. Newton's second law says:

$$\sum F_y = 0 \Rightarrow T_2 \sin 45^\circ = T_3 \Rightarrow T_2 = \frac{T_3}{\sin 45^\circ} \ .$$

But what's $T_3$? Well, there's another object (besides the knot) that $T_3$ acts on, and it's Block B. Here's the FBD for Block B:




Block B is also in equilibrium, so $\sum F_y = 0$. This implies that $T_3 = m_B g = (2.25 \text{ kg})(9.81 \text{ m/s}^2) = 22.07 \text{ N}$.
So:

$$T_2 = \frac{T_3}{\sin 45^\circ} = \frac{22.07 \text{ N}}{\sin 45^\circ} = 31.22 \text{ N} \ .$$

And: $$T_1 = T_2 \cos 45^\circ = (31.22 \text{ N}) \cos 45^\circ = 22.07 \text{ N} \ .$$

And finally, $$f_s = T_1 = 22.07 \text{ N} = 22.1 \text{ N} \ ,$$

keeping three significant figures.

(b.) If the mass of Block A is doubled, $f_s$ stays the same. Changing the mass of Block A doesn't do anything to $T_1$, and $f_s$ must always equal $T_1$ if Block A is to be in equilibrium.