Physics 1401
Homework Solutions - Walker, Chapter 4

Conceptual Questions

CE5. (a.) Given that the initial *speeds* are the same, we know that the magnitudes of the initial velocity vectors are the same. Therefore, the trajectory corresponding to the smallest horizontal component of initial velocity will be the steepest one. Similarly, the trajectory corresponding to the largest horizontal component of initial velocity will be the one that is least steep. So in order of increasing horizontal component of initial velocity, it is (a), (b), (c).

(b.) The trajectory with the longest time of flight will be the steepest one. The trajectory with the shortest time of flight will be the least steep one. Therefore, in order of increasing time of flight, it will be (c), (b), (a). As a check, we might derive an expression for the time when the projectile returns to its release point, taking the release point to be at $y_0 = 0$. The equation that gives *y* in terms of the time *t* is:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2,$$

taking the positive direction for *y* to be *upward*. If y_0 is taken to be zero, this becomes:

$$y = v_{y0}t - \frac{1}{2}gt^2.$$

Setting $y = 0$ and solving for *t*, we have:

$$0 = v_{y0}t - \frac{1}{2}gt^2$$

$$0 = t\left(v_{y0} - \frac{1}{2}gt\right)$$

There are two solutions, $t = 0$ (which is trivial) or $t = \frac{2v_{y0}}{g}$ (which is the interesting one). So the time of flight is greatest when v_{y0} is greatest, and smallest when v_{y0} is smallest. So it is as we concluded before: the steepest trajectory gives the greatest time of flight, and the least steep trajectory gives the smallest time of flight.

Problems

2. (a.) Think of the direction of due east as being the positive *x* direction and the direction of due north as being the positive *y* direction. Then your velocity in the direction of due east is $v_x = (1.60\ \text{m/s})\cos 15.0^\circ$. So how long does it take you to move 20.0 m in this direction? Well, taking your initial position to be $x_0 = 0, y_0 = 0$, your east-west position at any time *t* would be:

$$x = v_x t.$$

Solving for *t* gives:

$$t = \frac{x}{v_x}.$$

So to go a distance of 20.0 m east takes a time:

$$t = \frac{20.0\ \text{m}}{(1.60\ \text{m/s})\cos 15.0^\circ} = 12.9\ \text{s}$$

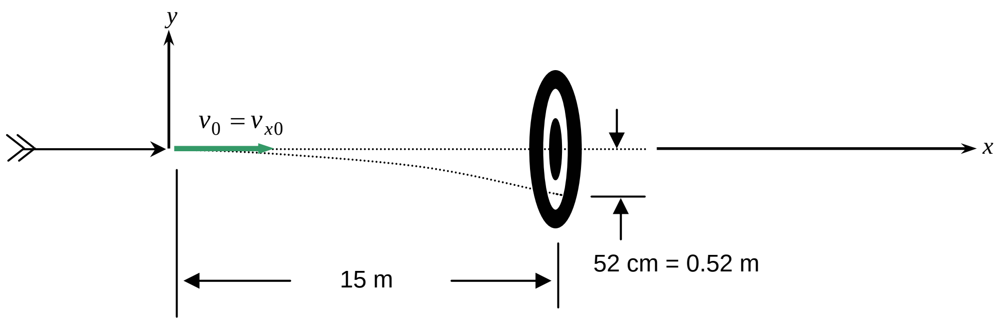
(b.) Your velocity in the direction of due north is $v_y = (1.60 \text{ m/s})\sin 15.0^\circ$. Your position in the direction of due north is:

$$y = v_y t\,.$$

Solving for t gives: $t = \dfrac{y}{v_y}$. So the time it takes you to go a distance of 30.0 m north is:

$$t = \frac{30.0 \text{ m}}{(1.60 \text{ m/s})\sin 15.0^\circ} = 72.4 \text{ s}$$

7. Picture:



The arrow drops vertically 52 cm (0.52 m) in the time it takes it to go 15 m *horizontally*. We would like to know: what does v_0 have to be in order for the arrow to go 15 m horizontally in the time it takes it to fall 0.52 m vertically? Well, how long does it take it to fall 0.52 m vertically? (This is like asking, "At what time t is y = -0.52 m?") (Note that I'm taking $x_0 = 0$ and $y_0 = 0$ here.) Well, the equation that gives y in terms of the time t is: $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$, or:

$$y = -\frac{1}{2}gt^2\,,$$

since $y_0 = 0$ and $v_{y0} = 0$. Setting y = -0.52 m and solving for t gives:

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-0.52 \text{ m})}{9.81 \text{ m/s}^2}} = 0.3256 \text{ s}$$

In this same time, the arrow travels 15 m horizontally (with *constant speed* v_0), so:

$$x = v_0 t\,.$$

Solving for v_0: $$v_0 = \frac{x}{t} = \frac{15\text{ m}}{0.3256\text{ s}} = 46\text{ m/s}$$

16. This one is very similar to Problem 7. We want to know: "What does v_0 have to be in order for x to be equal to 3.5 m when y is equal to –9.0 m?" (Note that I'm taking the initial position of the pumpkin to be $x_0 = 0,\ y_0 = 0$, the positive direction for x to be to the right, and the positive direction for y to be up.) Once again, the vertical position y is given as a function of the time t by:

$$y = -\frac{1}{2}gt^2 .$$

So the time when y = -9.0 m is:

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-9.0\text{ m})}{9.81\text{ m/s}^2}} = 1.355\text{ s} .$$

And the horizontal position is given by:

$$x = v_0 t .$$

(Note that the initial velocity is completely in the x direction.... The problem says the pumpkin is "thrown horizontally.") So we want x to equal 3.5 m when t = 1.355 s. In order for this to happen, we must have:

$$v_0 = \frac{3.5\text{ m}}{1.355\text{ s}} = 2.6\text{ m/s}$$

23. (a.) The horizontal component of the ball's velocity *does not change* while it is in flight. (Remember, a projectile has no acceleration in the horizontal direction while it is in flight.) So, just before the ball is caught, the horizontal component of its velocity will be whatever it was when it was released. We are told that it was released with a speed of 17.0 m/s at an angle of 35.0° above the horizontal. Therefore, the horizontal component of its velocity is initially:

$$v_{x0} = (17.0\text{ m/s})\cos 35.0^\circ$$

$$v_{x0} = 13.9\text{ m/s} .$$

This will be the horizontal component of the velocity just before the ball is caught.

(b.) This is like asking, "At what time t does the ball return to the vertical level at which it was released?" Taking this vertical level to be $y_0 = 0$, we want to find the time (after t = 0) when the ball returns to y = 0. Well, taking the positive direction for y to be upward, we have:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$y = v_{y0}t - \frac{1}{2}gt^2 .$$

Setting this equal to zero and solving for *t* gives two times, as we have seen before: $t = 0$ (trivial) and $t = \frac{2v_{y0}}{g}$ (the interesting one). So:

$$t = \frac{2(17.0 \text{ m/s})\sin 35.0^\circ}{9.81 \text{ m/s}^2} = 1.99 \text{ s} .$$

This is how long the ball is in the air.

29. (a.) It turns out that the speed on landing is the same, as long as the two balls have the same speed at the same height. This is a little subtle for right now, but will become much easier to see when we talk about the principle of **conservation of energy** in Chapter 8. In this problem, we have a ball that is thrown initially upward at an angle of 25° above the horizontal with a speed of 13 m/s. The ball is thrown from a height of 7.0 m above the ground. From the symmetry of projectile motion (in the absence of air resistance), when this ball returns to the height at which it was released (namely 7.0 m above the ground), it will have the same speed as when it was released (13 m/s). The ball thrown *straight down* also has a speed of 13 m/s at a height of 7.0 m above the ground. Therefore, the speeds of these two balls just before they hit the ground will be the same.

(b.) To verify the statement just made, let's calculate the speed of each ball just before it hits the ground. For the ball thrown straight down, we have (taking the positive direction for *y* to be up):

$$v^2 = v_0^2 - 2g(y - y_0) .$$

Let $y_0 = 0$. Then just before this ball hits the ground, *y* will equal -7.0 m. So the velocity *v* will be given by:

$$v^2 = (-13 \text{ m/s})^2 - 2(9.81 \text{ m/s}^2)(-7.0 \text{ m}) = 306.34 \text{ m}^2/\text{s}^2$$

In the final step, we must be careful. We know that the ball will be going *down* just before it hits. Therefore, with the choice of positive direction I have made, *v* should be negative. We have $v^2 = 306.34 \text{ m}^2/\text{s}^2$. When we take the square root of both sides to get *v*, we must remember that there are two possible answers, both of which give 306.34 when squared. These two possible answers are:

$$v = +\sqrt{306.34} \text{ m/s} \text{ and } v = -\sqrt{306.34} \text{ m/s} .$$

Clearly the one we want is the *negative* square root.

So: $v = -18 \text{ m/s}$,

keeping two significant figures. This means the *speed* (the *magnitude* of the velocity) is 18 m/s.

Now for the ball thrown up at an angle of 25° above the horizontal. The horizontal component of its velocity does not change. This horizontal component is:

$$v_x = v_{x0} = (13 \text{ m/s})\cos 25^\circ = 11.782 \text{ m/s} .$$

The vertical component of its velocity *does* change. We know that:

$$v_y^2 = v_{y0}^2 - 2g(y - y_0)$$

(taking the positive direction for y to be up.) So just before this ball hits the ground, its vertical component of velocity will be given by:

$$v_y^2 = [(13\text{ m/s})\sin 25^\circ]^2 - 2(9.81\text{ m/s}^2)(-7.0\text{ m}) = 167.524\text{ m}^2/\text{s}^2 .$$

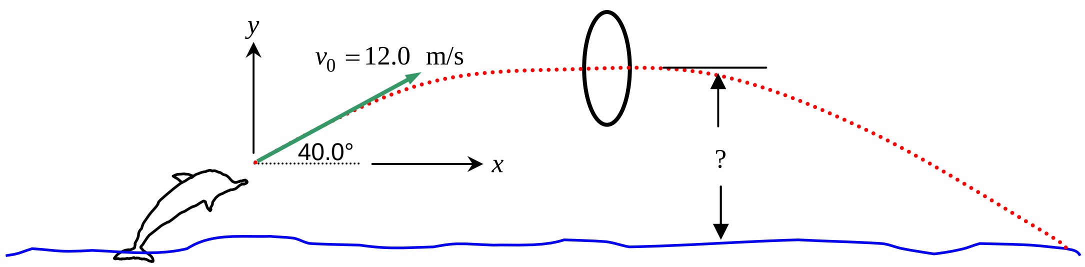
So: $$v_y = -12.943\text{ m/s} .$$

Finally, the *speed* of this ball just before it hits is the *magnitude* of its velocity just before it hits:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(11.782)^2 + (-12.943)^2} = 18\text{ m/s} ,$$

keeping two significant figures. So the speeds of the two balls are in fact the same just before they hit.

40. Picture:



If the dolphin is moving *horizontally* when it goes through the center of the hoop, then the y component of its velocity is zero at that moment in time. So we want to know, "What is y when $v_y = 0$?" Well, we know:

$$v_y = v_{y0} - gt .$$

Setting this equal to zero and solving for t, we get:

$$t = \frac{v_{y0}}{g} .$$

Now what is y at this time? Well,

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 .$$

So at $t = \frac{v_{y0}}{g}$: $$y = v_{y0}\left(\frac{v_{y0}}{g}\right) - \frac{1}{2}g\left(\frac{v_{y0}}{g}\right)^2 = \frac{v_{y0}^2}{2g} ,$$

taking $y_0 = 0$. So:

$$y = \frac{\left[(12.0 \text{ m/s})\sin 40.0^\circ\right]^2}{2\left(9.81 \text{ m/s}^2\right)} = 3.03 \text{ m} .$$

46. (a.) $v_x = v_{x0} = (10.2)\cos 25.0^\circ = 9.244 \text{ m/s}$. $v_{y0} = (10.2)\sin 25.0^\circ = 4.311 \text{ m/s}$. The vertical component of velocity at 0.250 s is:

$$v_y = v_{y0} - gt = 4.311 - (9.81)(0.250) = 1.858 \text{ m/s} .$$

So the magnitude of the velocity is:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.244)^2 + (1.858)^2} = 9.43 \text{ m/s} .$$

The angle that the velocity vector makes with the +x axis at t = 0.250 s is given by:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{1.858}{9.244}\right) = 11.4^\circ \text{ (above the horizontal).}$$

(b.)

$$v_y = v_{y0} - gt = 4.311 - (9.81)(0.500) = -0.594 \text{ m/s} .$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.244)^2 + (-0.594)^2} = 9.26 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-0.594}{9.244}\right) = -3.68^\circ \text{ (i.e., 3.68° } \textit{below} \text{ the horizontal).}$$

(c.) The ball is at its greatest height *before* 0.500 s, because at 0.500 s, the *y* component of its velocity is negative, meaning that it has already passed its highest point and has started on its way down.