Review of Basic Mathematics

Objective: To review and practice the math skills required for general chemistry.

Concepts: Basic mathematical formulas can be rearranged as necessary to solve problems. Data obtained from "word problems" can be used to solve mathematical problems encountered in chemistry. Significant figures minimize errors associated with measurements and calculations.

Text References: McMurray and Fay: Chapters 1.6-1.13

The Basic Math Review

You have been given a Math Test to assess your current level of math proficiency. It is essential that you have a good grasp of math and be able to handle normal algebraic rearrangement of formulas. If you had difficulty with the problems specifically listed as CHEM1411 level, you will have difficulty with this course. The following is a review of the basic mathematics required for this course. If you are not able to handle this level of math you probably should consider a refresher course in math before tackling CHEM 1411. CHEM1411 will be exponentially more difficult if you cannot do the math required.

Introduction

The ability to handle basic algebra problems will be directly related to your success or failure in chemistry. The mathematical manipulations that will be required to solve many of these problems are virtually identical to formula rearrangements you have performed in algebra. An understanding of geometry is also important but not as essential as your skill with algebra. You will also need to do simple conversions, the factor method or unit analysis, is the method of choice in these problems. Chemistry uses math as an essential tool.

Accuracy versus Precision

There is no such thing as a perfect measurement. Each measurement contains a degree of uncertainty due to the limits of the instruments and the people using them. In the lab, you are expected to follow the same procedures as chemists do when they make these same measurements. Each measurement should be reported as exactly as the device and your ability allow. This means that many of the digits in your measurement have a high degree of certainty and the last digit is estimated. Some **error** will always be introduced into every measurement you make. This is where the concepts of accuracy and precision come into play.

Many individuals use the words accuracy and precision interchangeably. In science the two words have significantly different meanings. **Accuracy** is defined as how close a reading is to the "actual" or accepted value. Accuracy cannot be discussed unless the absolute value is known. For example, if you shoot at a target, the distance that your shot hit from the bull's-eye exactly determines your accuracy. Because of this accuracy can be determined by a single measurement.

Precision is defined as how closely a group of measurements made with the same device and using the same level of care on the same object are to each other. Stated another way, precision refers to how close a series of measurements made on one object are to each other. Precision can only be determined after several repeated measurements are made.

It is possible to make an accurate measurement with an imprecise instrument and also to have a highly inaccurate measurement made with a highly precise device. In reality the actual definition of precision is probably closer to repeatability while accuracy can be defined as reliability.

Significant Figures

Because no measurement is perfect, you measurement expressed using ***significant figures***. This not only signals to others how reliable your number is, but it also minimizes the error associated with any calculation made with this measurement. Your calculator will mindlessly run the calculation made with your measurement and give you a stream of numbers for your answer – this answer will contain as many numbers as your calculator has available in its display. The majority of the numbers displayed in this answer are meaningless. The understanding of significant figures allows you to limit your answer to only those digits that are significant and allows you to eliminate those which compound error.

Certain assumptions are made when you take any measurement. Foremost of these assumptions is that the device you are using is accurate within the range of measurements that you are making. Therefore the numbers that you read from the device are relatively accurate. The error in this measurement is found in the very last digit either read from or estimated using the device. Since this very last digit is uncertain, it limits the level of certainty in your calculations. This is what limits the number of significant figures that you can have in your answer. In simplified terms, your answer is limited to the fewest number of significant figures present in the data from which you are making your calculation.

Rules for Determining the Number of Significant Figures

Because this is such an important part of experimental work, a set of rules has been established for determining the number of significant figures in a number. This is especially important when a zero is present in the reading.

Rule 1. Only the last digit of a recorded measurement is uncertain (estimated). All digits in this measurement are significant (including the last digit). All non-zero digits are automatically considered to be significant because it is assumed that these were read during the measurement. For example, 1.2543 has 5 significant figures.

Rule 2. Rules for reporting zeros:

- **a.** A zero between two non-zero integers is always significant. For example, 106 has 3 significant figures.
- **b.** Any zero that is not used to fix the position of a decimal point is significant. For example:
  - 50.10 has 4 significant figures
  - 1.00 has 3 significant figures
  - 6000. has 4 significant figures
- **c.** A zero is not significant if it merely fixes the position of the decimal point. For example:

6000 has 1 significant figure (the zeros are holding the decimal place)
0.000254 has 3 significant figures
The use of scientific notation would eliminate any confusion as to the number of significant figures present in a number. (It is automatically assumed that the individual who used scientific notation was following the rules for significant figures.)

6×10^7 has 1 significant figure
5.0×10^4 has 2 significant figures
4.200×10^3 has 4 significant figures

Be aware that the exponent in the scientific notation is not a significant number; it merely serves to hold the decimal place.

You should assume that the "trailing" zeros you encounter in most problems to be actual measurements rather than estimations. Therefore, most of these trailing zeros will be significant figures.

Rule 3. Certain numbers are exact, meaning that they have an infinite (unlimited) number of significant figures. For example 12 inches in 1 foot is an exact value. Both the 12 (inches) and 1 (foot) are exact values. So is 2.54 cm in 1 inch or 500 sheets in a ream of paper. Because these are defined values, they are exactly known and there is no uncertainty in their value. These values cannot be used to limit the number of significant figures in an answer.

Rules for Significant Figures in Calculations

The type of calculation must be considered when dealing with significant figures.

Rule 1. In addition and subtraction, the answer is expressed to the fewest number of decimal places found in the data. The decimal places establish the least reliable measurement (the least number of decimal places). For example:

Example a.	34.4356 g	*Example b.*	5.164 g
	105 g		5.05 g
	3.13 g		0.129 g
	143 g		13.34 g

In example a. 105 has no decimal points, the digit 5 indicates that this measurement was made to the nearest whole number; the answer cannot contain decimal places because 105 reflects the limit of precision/accuracy. In example b., 5.05 has three significant figures, and it indicates that the reliability of the measurement is to the nearest 0.01 g, the answer will be limited to two decimal places. In short the answer in addition and subtraction is limited to the fewest decimal places found in the values used for the calculation.

When performing additions or subtractions with exponential numbers it is best to first convert each of the numbers to the same exponential value. Then you can complete the addition or subtraction.

Rule 2. For multiplication and division, the answer cannot contain more significant figures than the value that has the fewest number of significant figures used in the calculation. For example, $2.00 \times 15.04 = 30.1$ (because 2.00 has 3 significant figures). Remember, defined values have unlimited numbers of significant figures. For example 12 inches × 2.54 cm/inch = 30.48 cm (no limit to significant figures since each of the values is exactly known).

Rounding Off

In general the rules you commonly use for rounding off numbers works the same in science. If a number is less than 5, it is dropped; if it is 5 or larger it is rounded up. A problem is encountered when the value is exactly 5. If you had a very large sample of numbers to round off, on average you would be changing values in the sample downwards 4/9ths of the time, compared to changing values in the sample upward 5/9ths of the time. This would mean that the average of the values AFTER rounding off would be greater than the average of the values BEFORE rounding. This is not acceptable.

The following rules dictate the manner in which numbers are to be rounded to the number of figures indicated. The first two rules are more-or-less the old ones. Rule three is the change in the old way to correct problems encountered when a value is exactly 5.

When rounding, examine the figure following (i.e., to the right of) the figure that is to be last. This figure you are examining is the first figure to be dropped.

1. If it is **less** than 5, drop it and all the figures to the right of it. (For example, 6.13 becomes 6.1)
2. If it is **more** than 5, increase the preceding figure by 1. (For example, 6.37 becomes 6.4)
3. If it **is** 5, round the preceding number so that it will be even. Keep in mind that zero is considered to be even when rounding off. (For example, 6.25 remains 6.2; 6.35 becomes 6.4)

Exponential Notation

Scientific notation, the use of exponents, is commonly encountered in chemistry. Significant figures follow a rather simple rule for scientific notation. This rule is that any integers (including zeros) in front of the exponent (i.e., $\times 10^x$) are significant. Your answer is limited to the fewest number of significant figures found in the data you used. For addition or subtraction of numbers that contain exponents you will first need to convert each value to a common exponent, perform the operation, and then convert back to the exponent that produces the desired number of significant figures.

Example. Add 1.23×10^6, 8.35×10^3 and 5.39×10^5.

First convert to a common exponent.

$$1.23 \times 10^6 = 12.3 \times 10^5$$
$$8.35 \times 10^3 = 0.0835 \times 10^5$$
$$5.39 \times 10^5 = 5.39 \times 10^5$$
$$17.7735 \times 10^5 = 1.78 \times 10^6$$

The exponent itself is not significant; it simply 'holds' the decimal place. When you multiply the exponents of the values are added together. When you divide, the exponents are subtracted.

> You may have encountered pH in relationship to the acidity of a material. The value stated as pH is a logarithm (actually negative log) of the concentration of the H^+ in solution. The first digit is actually the exponent of the value. Therefore when dealing with pH values, only the digits following the decimal place are "significant."

Rearranging Formulas

Mathematics is an essential part of chemistry. During the course of your study of chemistry, you will be required to memorize certain formulas and use them as necessary to solve problems and to calculate results from your experiments. Each formula is a template for you to use and modify as necessary depending upon the problem you are presented with. While it is possible to memorize every possible formula that you might be presented with, it is far easier to memorize a general formula and then rearrange it as necessary.

For example, moles are involved in the vast majority of problems that you will encounter. The formula used to calculate moles is:

$$\textbf{moles} = \frac{\textbf{weight of sample}}{\textbf{formula weight of compound}}$$

This formula can be rearranged to solve two other problems.

$$\textbf{weight of sample} = \textbf{moles} \times \textbf{formula weight of compound}$$

$$\textbf{formula weight of compound} = \frac{\textbf{weight of sample}}{\textbf{moles}}$$

While you could memorize all three formulas, it is easier to memorize the first and then rearrange it as needed. You will encounter this throughout the course. The harsh reality is that if you cannot handle the algebra required, you will have a difficult time in the course.

Percentages

You have dealt with percentages for years, now for a brief refresher on the subject. Percentages are representations of the fraction of a material found in another. This fraction is converted to a percentage by multiplying it by "100%." **To use this value in another calculation, you will need to convert it back to the decimal fraction** (or divide your answer by 100)**.**

First you must find the decimal fraction of the material of interest. This is done by dividing the mass (or quantity) of the material by the total mass (or quantity) of the sample it is in. This total mass includes the mass of the material you are interested in.

Example. What is the percentage of zinc (Zn) in zinc sulfide, ZnS? (atomic weight of Zn = 65.39, f.wt. ZnS = 97.46)

$$\text{Zn \%} = \frac{65.39 \text{ at. wt. Zn}}{97.46 \text{ f.wt ZnS}} \times 100\% = 0.6710 \times 100\% = 67.10\%$$

> Reminder, you must convert the % back to the decimal fraction before you use a percentage in a problem. Percentage is only one of the values based upon the fraction of weight of a material per given weight of sample. For instance, there is also a value based upon parts per thousand, ‰. Larger "parts per" values such as parts per million (ppm), parts per billion (ppb), and parts per trillion (ppt) are referred to with their abbreviations rather than with a symbol like % or ‰.

Calculations involving Squares and Square Roots
Although they may look imposing, problems that require manipulation of square roots are not that difficult if you remember one thing. Whatever you do to one side of the equal sign must be done to the other side as well. If you are solving for an unknown that is contained within a square root expression, square both sides of the equation to eliminate the square root. Then finish your manipulation of the equation and solve for the unknown.

Hints for Problems Involving Logarithms
You will encounter a few problems involving logarithms in CHEM 1411, you will see these with greater frequency in CHEM 1412. There are two varieties of logarithms and although they follow the same general rules for use and manipulation, they are distinctly different. Logarithms (base 10) show up in equations as simply log. If you have a logarithm (**log**), you convert this back to its original value by using the logarithm as a power of 10. The number in front of the decimal place will ultimately be the exponent when then number is expressed in scientific notation. The other logarithm you will encounter is a natural logarithm, or **ln**. To convert a value expressed as an **ln** back into a "real" number you will need to raise the value of e to that exponent, e^x. Your calculator has both **log** and **ln** values as well as 10^x and e^x functions. You must make certain that you use the correct logarithm.

Logarithms are technically "exponents" and as such, their use in multiplication and division work exactly like exponents. When logarithms are "multiplied" the log values are simply added together. Division with logarithms is simply subtraction. Since the advent of pocket calculators, logarithms are not commonly used outside of the sciences.

$$\ln \frac{P_1}{P_2} = \ln P_1 - \ln P_2$$

> **FYI.** Before the days of pocket calculators, aside from a slide rule, the best method for multiplication or division of large numbers was by use of logarithms. Primarily the logarithm tables that were available controlled the number of significant figures in your answer (generally between 3 and 5).

Word Problems

These are perhaps the most dreaded part of chemistry. These problems are much easier to solve if to approach them methodically. Before panicking when you are given word problem, first read it through carefully and decide exactly what is being asked for. Then go back through the problem and make a list of information that you are provided with (e.g. the compounds, their formulas, the weight, volume, and/or concentration of each, etc.). Once you have identified the data that is present, you can decide on the formula to use. Now plug the information you have listed from the problem into the formula and complete the calculation.

Care and Feeding of a Pocket Calculator

You will not be allowed to use a programmable calculator (e.g. TI-83, etc.) in the exams given for this course. Nor will you be allowed to use your cell phone as a calculator. It would be **best** if you practiced on homework and labs using a non-programmable calculator.

The calculators, regardless of manufacturer, have been assembled with certain "syntax" programmed into them. This means that when you enter data into the calculator and include function keys, the calculator will perform these functions in the order that **it was "programmed" to follow**. This may **not** be the order you intended when you plugged the numbers into the calculator. Sorry the calculator is permanently programmed into this order – you lose. Multiplication and division take precedence over addition and subtraction. You will need take this into account **or** use parentheses so that the calculator will group the numbers as you intended. **THIS TAKES PRACTICE** and an exam is the wrong time to "practice."