MEMO

TO: Your Project Team
FROM: Rob Eby, Senior Consultant for EB Enterprises
I am assigning your team the following project. And please, do not ask our client about his relatives and any experiments in old castles they might have been involved in.

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Mr Eby, I have been studying health for a while, and a fellow doctor told me about using a thing called matrices to model the spread of a virus. Can you help me with this? I have included a copy of what he sent me, but I could not figure it out. It has to do with how a virus spreads. I have my questions listed below. They crossed my mind while I was thinking about a new song I heard "Pudding on the Rich" or something like that.

Please explain your answers to the following questions:

1. How many second order contacts were there between the second contagious person (in the first group) and the first person in the third group?

2. Is there anyone in the third group who has had no contacts at all with the first group? If so, who and with whom?

3. Which person(s) in the third group had the most contacts, counting first and second-order contacts? How many of each, and with whom?

4. How many different people had second order-contacts, how did they have those contacts, and with whom?

Thanks for your help. Dr. Frank-en-steen
Some Help for You.

What follows is some information from Senior Consultant Eby to assist you in helping the client. And please, do not ask our client about his relatives and any experiments in old castles they might have been involved in.

Suppose that three people have contracted a contagious disease. A second group of five people may have been in contact with the three infected persons. A third group of six people may have been in contact with the second group. We can form a $3 \times 5$ matrix $R$ with the rows representing the first group of three, and the columns representing the second group of five. We enter a one in the corresponding position if a person in the first group has contact with a person in the second group. These direct contacts are called *first order contacts*. In a similar manner we form a $5 \times 6$ matrix $E$ representing the first order contacts between the second and third groups of people. For example, suppose

$$
R = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0
\end{bmatrix}
$$

$$
E = \begin{bmatrix}
1 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
$$

From $R$ we see that the first person in the first group had contact with the first and fourth persons in the second group. A *second order contact* is an indirect contact between persons in the first and third groups through some person in the second group. The product $RE$ indicates these contacts. Entry 2,4 in $RE$, that is $RE_{2,4} = 1$. This means there is one second order contact between the second person in group one and the fourth person in group three. To get this entry when multiplying, we did

$$
RE_{2,4} = R_{2,1}E_{1,4} + R_{2,2}E_{2,4} + R_{2,3}E_{3,4} + R_{2,4}E_{4,4} + R_{2,5}E_{5,4} = 0 \times 1 + 0 \times 0 + 1 \times 0 + 1 \times 1 + 0 \times 0 = 1
$$

The second person in group one and the fourth person in group three both had contact with the fourth person in group two.