Test #2
MATH 253

True or False (4 pts each)

T  F \[ \lim_{{(x,y) \to (1,0)}} \frac{x - y}{{x^2 - y^2}} = \frac{1}{2} \]

T  F If \( f(x, y) = \ln(y) \) then \( \nabla f(x, y) = \frac{1}{y} \)

T  F If \( f \) has a local maximum at \( (a, b) \) and \( f \) is differentiable at \( (a, b) \), then \( \langle f_x(a, b), f_y(a, b) \rangle = \langle 0, 0 \rangle \)

T  F If \( \vec{u} \cdot \nabla f = 0 \) then \( \vec{u} \) is tangent to a level curve of \( f \)

T  F \( D_{\vec{k}} f(x, y, z) = f_z(x, y, z) \)

\[ f(x, y, z) = x^2 y \ln yz \]

Find the indicated derivatives. (5 pts each)

\( f_x \)

\( f_y \)

\( f_{xz} \)

\( f_{zxy} \)
Show that \( u(x, t) = \sin(x - at) \) satisfies the wave equation \( \frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial t^2} \). (6 pts)

Find the derivative of \( f(x, y) = 2\sqrt{x} - y^2 \) at (1,5) in the direction from (3,4). (5 pts)

Find an equation of the tangent plane to \( z = x^2y + xy^3 \) at (1,1). (5 pts)
Find the points on the sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane $2x + y - 3z = 2$. (6 pts)

When is the directional derivative of $f$ at its maximum? (4 pts)

Find the domain and range for $f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2 - 1} + 4}$. (5 pts each)
Find the critical points of \( f(x, y) = x^2y - y^2x + 3xy \) and determine if they are saddle points, maximums or minimums. (12 pts)
A cylindrical fish tank is being built. The volume is to be 6434 cubic inches (about 50 gallons). The bottom of the tank is made of thicker stuff and costs 4 time as much as the side(s). The cost of the side(s) is 1 cent per square inch. Find the dimensions, in inches, of the tank that minimize the cost to build it. (12 pts)