True or False (4 pts each)

T  F  \[ \int_{x=0}^{2} \int_{y=0}^{\sqrt{4-x^2}} x^2y \, dy \, dx = \int_{r=0}^{2} \int_{\theta=0}^{\pi} r^4 \sin^2(\theta) \cos(\theta) \, d\theta \, dr \]

T  F  If \( f(x, y) \) is a continuous function in the plane then

\[ \int_{y=a}^{b} \int_{x=y}^{5} f(x, y) \, dx \, dy = \int_{x=a}^{b} \int_{y=x}^{5} f(x, y) \, dy \, dx \]

T  F  If \( D \) is the region with vertices \((0,0), (0,1), (1,0)\) then \( \iint_{D} 4 \, dA = 2 \)

T  F  \( \iint_{D} f(x, y) \, dx \, dy \) is the volume under the graph \( z = f(x, y) \)

Find the area outside the curve \( r = 1 + \cos(\theta) \) and inside the curve \( r = 3 \cos(\theta) \). (8 pts)

\[ \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_{r=1+\cos \theta}^{3 \cos \theta} r \, dr \, d\theta \]
I’ll leave the integration up to you. You should get a couple half angles. Be careful with the fractions.
Integrate the following. (6 pt ea)

\[
\int_{x=0}^{1} \int_{y=0}^{\sqrt{x}} y^2 e^{x^2} \, dy \, dx
\]

\[
\frac{e-1}{6}
\]

\[
\int_{x=0}^{3} \int_{y=0}^{x^2} x^2 y \, dy \, dx
\]

\[
\frac{3^{12}}{24}
\]

Set up the integral to find the volume under \( x^2 + z^2 + y^2 = 4 \) and above the cone \( z = \sqrt{4x^2 + 4y^2} \). (8 pts)

\[
\int_{r=0}^{2/\sqrt{3}} \int_{\theta=0}^{2\pi} \int_{z=2r}^{\sqrt{4-r^2}} r \, dz \, d\theta \, dr
\]
Set up the integral to find the surface area of that part of the sphere of radius 4 centered at the origin in the first octant above the cone $16z = \sqrt{x^2 + y^2}$. (8 pts)

The surface is $f(x, y) = \sqrt{16 - x^2 - y^2}$. The integral before picking a coordinate system is

$$\int_0^4 \int_0^{\frac{\pi}{2}} \int_0^{\tan^{-1}\left(\frac{1}{16}\right)} \sqrt{16 - x^2 - y^2} \, r^2 \sin \theta \, dr \, d\theta \, d\phi$$

In spherical coordinates $r = 4$, $\theta = 0.2\pi$, and $\phi = 0$.\tan^{-1}\left(\frac{1}{16}\right)$

Rewrite the integral $\int_{y=0}^{1} \int_{z=y^2}^{1} \int_{x=0}^{1-y} f(x, y, z) \, dx \, dz \, dy$ in the order $dz \, dy \, dx$. (8 pts)

$$\int_{x=0}^{1} \int_{y=0}^{1-x} \int_{z=y^2}^{1} f(x, y, z) \, dz \, dy \, dx$$
Set up the integrals to find the center of mass of the region bounded by the inside of $x^2 + y^2 = 2x$ and inside of $x^2 + y^2 = \frac{1}{2}$ where the density function is proportional to the distance from the intersection point of the two circles in the first quadrant. (12 pts)

The two circles intersect at $\left(\frac{1}{4}, \frac{\sqrt{7}}{4}\right)$. Therefore the density 

$$\rho(x, y) = \sqrt{(x - \frac{1}{4})^2 + (y - \frac{\sqrt{7}}{4})^2}.$$  The mass integral is

$$\int_{y=\frac{-\sqrt{2}}{4}}^{y=\frac{\sqrt{2}}{4}} \int_{x=1-\sqrt{1-y^2}}^{x=\frac{1}{2}-\sqrt{\frac{1}{2}-y^2}} \rho(x, y) \, dx \, dy.$$  The other integrals are similar.
Find the volume of the solid cut from the sphere of radius 2 (centered at the origin) by the planes $x = y$ and $x = 2y$. (8 pts)
R is the region bounded by $y = x, y = x - 2, y = -2x, y = 3 - 2x$ (see graph below). Apply the transformation $x = \frac{1}{3}(u + v), y = \frac{1}{3}(u - 2v)$ and graph the resulting figure label the lines and the corners and where they end up. (8 pts)

Replace the $x,y$ in each equation and simplify. Plot each new equation and label.
Find the Jacobian of the transformation above. (6 pts)

\[ \begin{vmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} \end{vmatrix} = \frac{3}{9} \] The jacobian is the absolute value of the determinant.

Use the previous information to integrate \( \int_{R} \int xy \, dA \). (6 pts)