Find an equation for the sphere with the given center and radius.

1) Center (0, 0, 5), radius = 6

Find the angle between \( u \) and \( v \) in radians.

2) \( u = 3j - 6k, v = 6i - 9j - 6k \)

Find an equation for the line that passes through the given point and satisfies the given conditions.

3) \( P = (-8, -5) \); perpendicular to \( v = -6i + 9j \)

Find the vector \( \text{proj}_v u \).

4) \( v = 7i - 3j + k, \ u = -4j + 3k \)

Find the angle between the planes.

5) \( 8x + 3y + 4z = -3 \) and \( 2x + 9y + 8z = -1 \)

Calculate the requested distance.

6) The distance from the point \( S(7, 1, -5) \) to the line \( x = -6 + 3t, y = -9 + 12t, z = 10 + 4t \)

7) The distance from the point \( S(5, 2, 3) \) to the plane \( 2x + 2y + z = 7 \)

Write the equation for the plane.

8) The plane through the point \( P(-10, -7, 9) \) and perpendicular to the line \( x = 7 + 7t, y = -5 + 9t, z = 7 - t \).

Find parametric equations for the line described below.

9) The line through the point \( P(-1, 2, 0) \) and perpendicular to the plane \( 4x + 6y + 4z = 5 \)

The vector \( r(t) \) is the position vector of a particle at time \( t \). Find the angle between the velocity and the acceleration vectors at time \( t = 0 \).

10) \( r(t) = e^{7t}i + (9 + e^{-7t})j + (5 \cos 3t)k \)

If \( r(t) \) is the position vector of a particle in the plane at time \( t \), find the indicated vector.

11) \( r(t) = (-5t^2 - 9)i \star \left( \frac{1}{15} \right)^3 j \).

Find the velocity vector of the particle.

The position vector of a particle is \( r(t) \). Find the requested vector.

12) The velocity at \( t = 0 \) for \( r(t) = \cos(2t)i + 9\ln(t - 3)j - \frac{t^3}{3}k \)

Calculate the arc length of the indicated portion of the curve \( r(t) \).

13) \( r(t) = (7 - 2t)i + (5 + 6t)j + (9t - 7)k, -10 \leq t \leq -3 \)

Find the length of the indicated portion of the curve.

14) \( r(t) = (e^{\cos t})i + (e^{\sin t})j + 5e^{4t}k, -\ln 2 \leq t \leq 0 \)

Find the unit tangent vector of the given curve.

15) \( r(t) = (8 \sin^3 5t)i + (8 \cos^3 5t)j \)

Find the curvature of the curve \( r(t) \).

16) \( r(t) = (3 + \cos 4t - \sin 4t)i + (7 + \sin 4t + \cos 4t)j + 3k \)

Find the curvature of the space curve.

17) \( r(t) = -7i + (t + 2)j + (\ln(\cos t) + 10)k \)

Find the principal unit normal vector \( N \) for the curve \( r(t) \).

18) \( r(t) = (9 + t)i + (2 + \ln(\cos t))k, -\pi/2 < t < \pi/2 \)

For the curve \( r(t) \), write the acceleration in the form \( a = \alpha N \).

19) \( r(t) = 4(1 + t)^{3/2}i + 4(1 - t)^{3/2}j + 3tk \)

Find two paths of approach from which one can conclude that the function has no limit as \( (x, y) \) approaches \( (0, 0) \).

20) \( f(x, y) = \frac{x^2y}{x^2 + y^2} \)

Find the limit.

21) \( \lim_{(x, y) \to (2, 2)} e^{-x^2 - y^2} \)

Determine whether the given function satisfies Laplace’s equation.

22) \( f(x, y, z) = \cos (8x) \sin (8y) e^{(\sqrt{128}z)} \)

Determine whether the given function satisfies the wave equation.

23) \( w(x, t) = \sin (5x + 5ct) \)
Find all the first order partial derivatives for the following function.

24) \( f(x, y, z) = \frac{\cos y}{xz^2} \)

Find all the second order partial derivatives of the given function.

25) \( f(x, y) = \cos xy^2 \)

Solve the problem.

26) Evaluate \( \frac{\partial w}{\partial u} \) at \( (u, v) = (1, 4) \) for the function \( w(x, y) = xy - y^2; x = u - v, \ y = uv. \)

Use implicit differentiation to find the specified derivative at the given point.

27) Find \( \frac{\partial x}{\partial y} \) at the point \( \left(1, \frac{\pi}{28}\right) \) for \( e^{x^2\cos yz} = 0. \)

Compute the gradient of the function at the given point.

28) \( f(x, y, z) = \ln(x^2 + 3y^2 + 2z^2); (3, 3, 3) \)

Answer the question.

29) Find the direction in which the function is increasing or decreasing most rapidly at the point \( P_0 \).

\( f(x, y, z) = x\sqrt{y^2 + z^2}; P_0(1, 2, 1) \)

Find the derivative of the function at the given point in the direction of \( A. \)

30) \( f(x, y, z) = -3x + 6y - 9z; (-9, 10, -2) \)

\( A = 3i - 6j - 2k \)

Solve the problem.

31) Write an equation for the tangent line to the curve \( x^2 - 2xy + y^2 = 4 \) at the point \( (-1, 1). \)

32) Find parametric equations for the normal line to the surface \( z = \ln(8x^2 + 3y^2 + 1) \) at the point \( (0, 0, 0). \)

33) Find the equation for the tangent plane to the surface \( z = e^{9x^2 + 4y^2} \) at the point \( (0, 0, 1). \)

Find the extreme values of the function subject to the given constraint.

34) \( f(x, y) = x^2 + y^2; \ xy^2 = 128 \)

Determine the order of integration and then evaluate the integral.

35) \( \int_0^6 \int_x^6 \frac{\sin y}{y} \, dy \, dx \)

36) \( \int_0^2 \int_{\sqrt{x}/2}^1 e^{y^3} \, dy \, dx \)

Find the volume of the indicated region.

37) The region bounded by the paraboloid \( z = 36 - x^2 - y^2 \) and the xy-plane

38) The solid cut from the first octant by the surface \( z = 25 - x^2 - y \)

Integrate the function \( f \) over the given region.

39) \( f(x, y) = \frac{1}{xy} \) over the square \( 6 \leq x \leq 8, 6 \leq y \leq 8 \)

40) \( f(x, y) = \frac{x}{3} + \frac{y}{6} \) over the trapezoidal region bounded by the x-axis, y-axis, line \( x = 3 \), and line \( y = -\frac{4}{3}x + 10 \)

Write an equivalent double integral with the order of integration reversed.

41) \( \int_0^{\pi/2} \int_0^\infty \sin x \ (10x + 8y) \, dy \, dx \)

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

42) The curves \( y = x(x - 6) \) and \( y = x(x - 6)(x - 10) \)

43) The lines \( x = 0, y = 3x, \) and \( y = 5 \)
Change the Cartesian integral to an equivalent polar integral, and then evaluate.

\[ \int_{0}^{11} \int_{0}^{\sqrt{121 - y^2}} (x^2 + y^2) \, dx \, dy \]

Find the area of the region specified in polar coordinates.

45) One petal of the rose curve \( r = 7 \sin \theta \)

Evaluate the integral by changing the order of integration in an appropriate way.

46) \[ \int_{0}^{4} \int_{y}^{4} \int_{0}^{\pi} \sin z \sin x \frac{dz \, dx \, dy}{x} \]

Evaluate the integral.

47) \[ \int_{0}^{2} \int_{0}^{2r} \int_{0}^{\pi/2} r \, d\theta \, dr \, dz \]

Evaluate the cylindrical coordinate integral.

49) \[ \int_{0}^{\pi/2} \int_{0}^{\pi/2} \int_{0}^{\pi/2} \rho^3 \sin \phi \, d\phi \, d\rho \, d\theta \]

Evaluate the spherical coordinate integral.

51) \[ \int_{0}^{\pi/2} \int_{0}^{\pi/2} \int_{0}^{\pi/6} \rho \, d\phi \, d\rho \, d\theta \]

Find the volume of the indicated region.

52) The region enclosed by the sphere \( x^2 + y^2 + z^2 = 64 \) and the cylinder \( (x - 4)^2 + y^2 = 16 \)

53) The region bounded above by the sphere \( x^2 + y^2 + z^2 = 36 \) and below by the cone \( z = \sqrt{x^2 + y^2} \)

Set up the iterated integral for evaluating

\[ \int \int \int f(r, \theta, z) \, dz \, r \, dr \, d\theta \]

over the given region D.

54) D is the right circular cylinder whose base is the circle \( r = 4 \cos \theta \) in the xy-plane and whose top lies in the plane \( z = 10 - x - y \).

55) D is the solid right cylinder whose base is the region in the xy-plane that lies inside the cardioid \( r = 10 + 3 \cos \theta \) and outside the circle \( r = 7 \), and whose top lies in the plane \( z = 9 \).

Use the given transformation to evaluate the integral.

56) \[ u = -4x + y, \quad v = 2x + y; \]

\[ \int \int (2x + y) \, dx \, dy, \]

where \( R \) is the parallelogram bounded by the lines \( y = 4x + 4, \ y = 4x + 7, \ y = -2x + 2, \ y = -2x + 8 \)

Find the Jacobian \( \frac{\partial(x, y)}{\partial(u, v)} \) or \( \frac{\partial(x, y, z)}{\partial(u, v, w)} \) (as appropriate)

using the given equations.

57) \( x = 3u^2, \ y = 10uv \)

Evaluate the line integral of \( f(x, y) \) along the curve \( C \).

58) \( f(x, y) = \cos x + \sin y, \ C: \ y = x, \ 0 \leq x \leq \frac{\pi}{2} \)

Evaluate the line integral along the curve \( C \).

59) \[ \int_{C} \frac{x + y + z}{5} \, ds, \ C \ is \ the \ curve \ \mathbf{r}(t) = 3t \mathbf{i} + (8 \cos \frac{1}{2}t) \mathbf{j} + (8 \sin \frac{1}{2}t) \mathbf{k}, \ 0 \leq t \leq 2\pi \]

Find the mass of the wire that lies along the curve \( r \) and has density \( \delta \).

60) \( r(t) = (7 \cos t) \mathbf{i} + (7 \sin t) \mathbf{j} + 7t \mathbf{k}, \ 0 \leq t \leq 2\pi; \ \delta = \frac{8}{\pi} \)
Calculate the flux of the field \( F \) across the closed plane curve \( C \).

61) \( F = (x+y)i + xyj; \) the curve \( C \) is the closed counterclockwise path around the rectangle with vertices at \((0, 0), (7, 0), (7, 6), \) and \((0, 6)\)

Calculate the work done by the force \( F \) along the path \( C \).

62) \( F = -5zi + 6xj + 7yk; \) \( C: r(t) = ti + tj + tk, \) \( 0 \leq t \leq 1 \)

Find the gradient field \( F \) of the function \( f \).

63) \( f(x, y, z) = \frac{x^2 + y^2 + z^2}{x^7} \)

Find the potential function \( f \) for the field \( F \).

64) \( F = 4x^3y^3z^3 + 10x^4y^3z^3j + 8x^2y^3z^3k \)

Using Green’s Theorem, calculate the area of the indicated region.

65) The circle \( r(t) = (7 \cos t)i + (7 \sin t)j, \) \( 0 \leq t \leq 2 \pi \)

66) The area bounded above by \( y = 7 \) and below by \( y = \frac{7}{81}x^2 \)

Apply Green’s Theorem to evaluate the integral.

67) \( \oint_C (8y + x) \, dx + (y + 3x) \, dy \)

\( C: \) The circle \((x - 4)^2 + (y - 6)^2 = 16\)

68) \( \oint_C (y^2 + 5) \, dx + (x^2 + 1) \, dy \)

\( C: \) The triangle bounded by \( x = 0, x + y = 1, y = 0 \)

Using Green’s Theorem, find the outward flux of \( F \) across the closed curve \( C \).

69) \( F = (-y - e^y \cos x)i + (y - e^y \sin x)j; \) \( C \) is the right lobe of the lemniscate \( r^2 = \cos 2\theta \) that lies in the first quadrant.

Evaluate the surface integral of the function \( g \) over the surface \( S \).

70) \( g(x,y,z) = x^2 + y^2 + z^2; \) \( S \) is the surface of the cube formed from the coordinate planes and the planes \( x = 1, y = 1, \) and \( z = 1 \)

Find the flux of the vector field \( F \) across the surface \( S \) in the indicated direction.

71) \( F = 4i + y^2j - y^2z^2k, \) \( S \) is the rectangular surface \( x = 0, -6 \leq y \leq 6, \) and \(-8 \leq z \leq 8, \) direction \( i \)

Find the surface area of the surface \( S \).

72) \( S \) is the portion of the surface \( 3x + 4z = 4 \) that lies above the rectangle \( 7 \leq x \leq 10 \) and \( 4 \leq y \leq 8 \) in the \( x-y \) plane

73) \( S \) is the portion of the paraboloid \( z = 4 - x^2 - y^2 \) that lies above the ring \( 4 \leq x^2 + y^2 \leq 9 \) in the \( x-y \) plane.

Evaluate the surface integral of \( g \) over the surface \( S \).

74) \( S \) is the plane \( x + y + z = 3 \) above the rectangle \( 0 \leq x \leq 5 \) and \( 0 \leq y \leq 2; \) \( g(x,y,z) = 3z \)

75) \( S \) is the parabolic cylinder \( y = 5x^2, \) \( 0 \leq x \leq 4 \) and \( 0 \leq z \leq 3; \) \( g(x, y, z) = 3x \)

Find the flux of the vector field \( F \) across the surface \( S \) in the indicated direction.

76) \( F = 8xi + 8yj + zk; \) \( S \) is portion of the plane \( x + y + z = 5 \) for which \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq 4; \)

direction is outward (away from origin)

77) \( F = x^3y^4i + yj - zk; \) \( S \) is the portion of the parabolic cylinder \( z = 1 - y^2 \) for which \( z \geq 0 \) and \( 2 \leq x \leq 3; \) direction is outward (away from the \( x-y \) plane)

Calculate the area of the surface \( S \).

78) \( S \) is the portion of the sphere \( x^2 + y^2 + z^2 = 81 \)

between \( z = -\frac{9}{2} \sqrt{2} \) and \( z = \frac{9}{2} \sqrt{2} \)

79) \( S \) is the cap cut from the paraboloid \( z = \frac{9}{20} - 5 \)

between \( x^2 - 5y^2 \) by the cone \( z = \sqrt{x^2 + y^2} \)
Use Stokes' Theorem to calculate the circulation of the field $F$ around the curve $C$ in the indicated direction.

80) $F = 5yi + 4xj - 2z^3k$; $C$: the portion of the plane $2x + 3y + 9z = 4$ in the first quadrant.

81) $F = 3xi + 2xj + 7zk$; $C$: the cap cut from the upper hemisphere $x^2 + y^2 + z^2 = 16$ ($z \geq 0$) by the cylinder $x^2 + y^2 = 4$. 

Answer Key
Testname: CAL3-FINAL-PT

1) \(x^2 + y^2 + z^2 - 10z = 11\)
2) 1.46
3) \(-6x + 9y = 3\)
4) \(\frac{10}{59}i - \frac{45}{59}j + \frac{15}{59}k\)
5) 0.862
6) \(\sqrt[3]{73.685}/13\)
7) \(\frac{10}{3}\)
8) \(7x + 9y - z = -142\)
9) \(x = 4t - 1, y = 6t + 2, z = 4t\)
10) \(\frac{\pi}{2}\)
11) \(v(t) = (-10t)i + \left(\frac{1}{5}t^3\right)j\)
12) \(v(0) = -3j\)
13) 77
14) \(\frac{3\sqrt{3}}{2}\)
15) \(T(t) = (\sin 5t)i - (\cos 5t)j\)
16) \(\kappa = \frac{\sqrt{2}}{2}\)
17) \(\kappa = \cos t\)
18) \(N(t) = (-\sin t)i - (\cos t)k\)
19) \(a = 3\left[\frac{2}{\sqrt{1 - t^2}}N\right]\)
20) Answers will vary. One possibility is Path 1: \(x = t, y = t\); Path 2: \(x = t, y = t^2\)
21) \(e^{-8}\)
22) Yes
23) Yes
24) \(\frac{\partial f}{\partial x} = -\frac{\cos y}{x^2z^2}, \frac{\partial f}{\partial y} = -\frac{\sin y}{xz^2}, \frac{\partial f}{\partial z} = -\frac{2 \cos y}{xz}\)
25) \(\frac{\partial^2 f}{\partial x^2} = -y^4 \cos xy^2, \frac{\partial^2 f}{\partial y^2} = -2x[2xy^2 \cos(xy^2) + \sin(x y^2)]; \frac{\partial^2 f}{\partial y\partial x} = \frac{\partial^2 f}{\partial x\partial y} = -2y[xy^2 \cos(xy^2) + \sin(xy^2)];\)
26) -40
27) \(\frac{7}{2}\)
28) \(\frac{1}{9}i + \frac{1}{3}j + \frac{2}{9}k\)
29) \(\sqrt{5}\left[\frac{1}{15} + \frac{j}{15} + \frac{k}{30}\right]\)
30) \(-\frac{27}{7}\)
31) \(x - y + 2 = 0\)
32) \(x=0, y=0, z=t\)
33) \(z = 1\)
34) Maximum: none; minimum: \(48\) at \((4, \pm 4\sqrt{2})\)
35) \(1 - \cos 6\)
36) \(\frac{2}{3}(e - 1)\)
37) \(648\pi\)
38) \(\frac{2500}{3}\)
39) \(\left[\ln\left(\frac{3}{4}\right)^2\right]\)
40) \(\frac{82}{3}\)
41) \(\int_1^\pi \int_0^{\pi/2} (10x + 8y) \, dx \, dy\)
42) \(\int_6^\infty \int_0^x (x - 6)(x - 10) \, dy \, dx\)
43) \(\int_0^5 \int_0^ {y/2} \, dx \, dy\)
44) \(\frac{14,641\pi}{8}\)
45) \(\frac{49}{8}\pi\)
46) \(\frac{\ln^2 1297}{4}\)
47) \(2(1 - \cos 4)\)
48) 868
49) \(480\pi\)
50) \(\frac{512 \ln 2 - 3}{1024}\)
51) \(7\pi\)
52) \(\frac{1024}{9}(3\pi - 4)\)
53) \(72\pi(2 - \sqrt{2})\)
54) \(\int_0^\pi \int_0^4 \int_0^{10 - r} f(r, \theta, z) \, dz \, dr \, \theta\)
55) \(\int_0^{2\pi} \int_0^7 \int_0^9 f(r, \theta, z) \, dz \, dr \, d\theta\)
56) 15
57) 60u^2
58) 2\sqrt{2}
59) 6\pi^2 + 32
60) 112\pi\sqrt{2} \text{ units}
61) 189
62) W = 4
63) \mathbf{F} = \frac{-5x^2 - 7y^2 - 7z^2}{x^8} \mathbf{i} + \frac{2y}{x^7} \mathbf{j} + \frac{2z}{x^7} \mathbf{k}
64) f(x, y, z) = x^4y^{10}z^8 + C
65) 49\pi
66) 84
67) -80\pi
68) 0
69) \frac{1}{2}
70) 7
71) 768
72) 15
73) \frac{\pi}{6}[37\sqrt{37} - 17\sqrt{17}]
74) -15\sqrt{3}
75) \frac{3}{100}(1601\sqrt{1601} - 1)
76) 90
77) 0
78) 81\sqrt{2}\pi
79) \frac{\pi}{150}[2\sqrt{2} - 1]
80) -\frac{4}{3}
81) 8\pi