Math 2414 Take home Questions Exam 4 Due at the end of the review day

Group Members Names

1. For each of the five definite integrals given below, explain which ones are improper and why. For each of them, compute the value of the definite integral like you would on the in class part of the exam.
   - I $\int_1^{\infty} \frac{\sin x}{x}\,dx$
   - II $\int_4^5 \frac{1}{x}\,dx$
   - III $\int_3^4 \frac{dx}{\sin x}$
   - IV $\int_{-3}^{3} x^{-1/3}\,dx$
   - V $\int_{-10}^{10} f(x)dx$ where $f(x) = 2 + 1/x, \quad -10 \le x < -1 \qquad f(x) = \frac{1}{x+2}, \quad -1 \le x \le 10$

2. Write each of the following in summation notation, then write the definite integral that goes with it.
   - (I) $\lim\limits_{n\to\infty} \frac{1}{n}\left[(\frac{1}{n})^3 + (\frac{2}{n})^3 + \cdots + (\frac{n}{n})^3\right]$ (II) $\lim\limits_{n\to\infty} \frac{1}{n}\left[(\frac{1^5}{n^5}) + (\frac{2^5}{n^5}) + \cdots + (\frac{n^5}{n^5})\right]$
   - (III) $\lim\limits_{n\to\infty} \frac{7}{n}\left[(\frac{n+7}{n})^3 + (\frac{n+14}{n})^3 + (\frac{n+21}{n})^3 + \cdots + (\frac{n+7n}{n})^3\right]$
   - (IV) $\lim\limits_{n\to\infty} \frac{42}{n}\left[(\frac{2n+42}{n})^5 + (\frac{2n+84}{n})^5 + (\frac{2n+126}{n})^5 + \cdots + (\frac{2n+42n}{n})^5\right]$
   - (R_1) $\lim\limits_{n\to\infty} \frac{1}{n}\left[\frac{n}{n+1} + \frac{n}{n+2} + \frac{n}{n+3} + \frac{n}{n+4} + \cdots + \frac{n}{n+n}\right]$ (R_2) $\lim\limits_{n\to\infty} \left[\frac{1}{1+1} + \frac{1}{1+2} + \frac{1}{1+3} + \frac{1}{1+4} + \frac{1}{1+5} + \cdots + \frac{1}{2+n}\right]$
   - (V) $\lim\limits_{n\to\infty} \frac{n+1}{n}\left[(\frac{n+1}{n})^3 + (\frac{n+2}{n})^3 + \cdots + (\frac{2n}{n})^3\right]$

3. Find the sums of each of the following series. Make note of any patterns you see.

$$S_1 = \sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+3}}\right) \quad S_2 = \sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+4}}\right)$$

$$S_3 = \sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+5}}\right) \quad S_4 = \sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+6}}\right)$$

4. Find the sums of each of the following series. Make note of any patterns you see.

$$S_1 = \sum_{k=1}^{\infty}\frac{1}{k^2+2k} \qquad S_2 = \sum_{k=1}^{\infty}\frac{1}{k^2+3k} \qquad S_3 = \sum_{k=1}^{\infty}\frac{1}{k^2+5k} \qquad S_4 = \sum_{k=1}^{\infty}\frac{1}{k^2+8k}$$

5. Label the following are always true, sometimes true, or never true. Either provide a proof or for the sometimes, give two examples, one showing the truth, one showing the non-truth.
   - a If $\sum r_n$ converges, then $r_n \to 0$.
   - b If $r_n \to 0$, then $\sum r_n$ converges.
   - c $\sum\limits_{k=0}^{\infty} x^k = \frac{1}{1-x}$ whenever $x \neq 1$.
   - d If $\sum E_n$ and $\sum B_n$ each diverge, then $\sum E_N + B_N$ diverges.
   - e If $\sum E_n$ converges and $\sum B_n$ diverges, then $\sum E_N + B_N$ diverges.
   - f If $\{r_n\}$ is monotonic and bounded, then $\sum r_n$ converges.
   - g If the partial sums of $\sum r_n$ are bounded, then $\sum r_n$ converges.
   - h If $r_n \ge 0$ and the partial sums of $\sum r_n$ are bounded, then $\sum r_n$ converges.
   - i If $\sum E_n$ and $\sum B_n$ each converge, then $\sum (E_N * B_N) = (\sum E_N) * (\sum B_n)$.