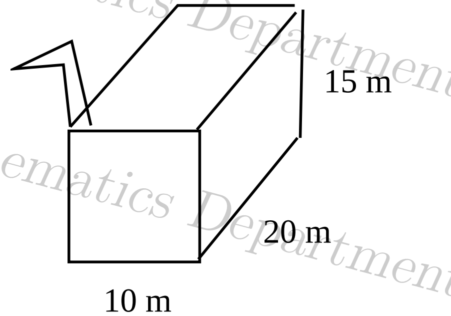
Math 2414 Take home Questions Exam 3 Due at the end of class Wednesday

Group Members Names

1. (30 points) Compute the antiderivative of each of the following functions. Do you notice any patterns in the answers? HINT: It might help to (1) get everything over a common denominator (2) not reduce fractions once you combine terms, but instead try to see patterns in the numbers (3) keep the $+C$ by itself.
   - $f_1(x) = \ln(x)$
   - $f_2(x) = x\ln(x)$
   - $f_3(x) = x^2\ln(x)$
   - $f_4(x) = x^3\ln(x)$
   - $f_5(x) = x^4\ln(x)$
   - $f_6(x) = x^5\ln(x)$
2. (30 points) There is a window in a dam and the center (incenter) of the window is 10 m below the surface of the water. Find the hydrostatic force upon the window in each of the following cases, or explain why you cannot find it in the particular instance.
   - i The window is a square of area $25m^2$.
   - ii The window is a circle of area $25m^2$.
   - iii The window is a rectangle of area $25m^2$.
   - iv The window is an isosceles triangle of area $25m^2$.
   - v The window is an equilateral triangle of area $25m^2$.
3. (30 points) Write each of the following into partial fractions of two fractions total. Be sure to make mention of any patterns you find.
   - I $\frac{1}{x(x-1)}$
   - II $\frac{1}{x(x-2)}$
   - III $\frac{1}{x(x-3)}$
   - IV $\frac{1}{x(x-4)}$

4. (30 points) We have a tank in the shape of a rectangle that is $10m$ by $20m$ and $15m$ high. The outlet spout is $8m$ above the top of the tank. For each of the integrals below (A-F), MATCH the description (I - VI) of where the axis was centered, and in which direction (up / down) is the positive y with the integral.



$$(A)\int_0^{15} 1000(9.80)(y+8)(10)(20)dy \qquad (B)\int_{-15}^{0} 1000(9.80)(8-y)(10)(20)dy \qquad (C)\int_0^{15} 1000(9.80)(y+8)(10)(20)dy$$

$$(D)\int_{-23}^{-8} 1000(9.80)(-y)(10)(20)dy \qquad (E)\int_{-2}^{13} 1000(9.80)(21-y)(10)(20)dy \qquad (F)\int_{-17}^{-2} 1000(9.80)(6-y)(10)(20)dy$$

I At the top of the pipe, up

II $2m$ up from the bottom middle of the box, up

III At the top middle of the box, down

IV At the bottom middle of the box, up

V $6m$ down from the top of the pipe, up

VI At the top middle of the box, up

5. (30 points) We have four different water tanks, each $42m$ long, and each has a spout $8m$ above the top of the tank through which we will pump out the water. However,
(A) has an inverted isosceles triangle $10m$ wide and $10m$ high for its ends,
(B) has an upright isosceles triangle $10m$ wide and $10m$ high for its ends,
(C) has the lower half of a semi-circle of radius $10m$ for its ends, and
(D) has the upper half of a semi-circle of radius $10m$ for its ends.

Match each of the eight integrals below with the correct face and location of the axis for determining the total work needed to pump the water out of the tank.

(i) A, bottom, up (ii) B, bottom, up (iii) C, bottom, up (iv) D, bottom, up
(v) A, top, up (vi) B, top, up (vii) C, top, up (viii) D, top, up

$$(I)\int_0^{10} 1000(9.80)(18-y)(42)\left(2\frac{y}{2}\right)dy \qquad (II)\int_0^{10} 1000(9.80)(18-y)(42)\left(2\frac{10-y}{2}\right)dy$$

$$(III)\int_0^{10} 1000(9.80)(18-y)(42)(2\sqrt{10^2-(y-10)^2})dy \qquad (IV)\int_0^{10} 1000(9.80)(18-y)(42)(2\sqrt{10^2-y^2})dy$$

$$(V)\int_{-10}^{0} 1000(9.80)(8-y)(42)\left(2\frac{10+y}{2}\right)dy \qquad (VI)\int_{-10}^{0} 1000(9.80)(8-y)(42)\left(2\frac{y}{2}\right)dy$$

$$(VII)\int_{-10}^{0} 1000(9.80)(8-y)(42)(2\sqrt{10^2-y^2})dy \qquad (VIII)\int_{-10}^{0} 1000(9.80)(8-y)(42)(2\sqrt{10^2-(y+10)^2})dy$$