

Math 2414 Take home Questions Exam 3 ANSWERS

1. (30 points) Compute the antiderivative of each of the following functions. Do you notice any patterns in the answers? HINT: It might help to (1) get everything over a common denominator (2) not reduce fractions once you combine terms, but instead try to see patterns in the numbers (3) keep the $+C$ by itself.

- $f_1(x) = \ln(x) \quad \int f_1(x)dx = x \ln(x) - x + C$
- $f_2(x) = x \ln(x) \quad \int f_2(x)dx = \frac{x^2}{2} \ln(x) - \frac{x^2}{4} + C$
- $f_3(x) = x^2 \ln(x) \quad \int f_3(x)dx = \frac{x^3}{3} \ln(x) - \frac{x^3}{9} + C$
- $f_4(x) = x^3 \ln(x) \quad \int f_4(x)dx = \frac{x^4}{4} \ln(x) - \frac{x^4}{16} + C$
- $f_5(x) = x^4 \ln(x) \quad \int f_5(x)dx = \frac{x^5}{5} \ln(x) - \frac{x^5}{25} + C$
- $f_6(x) = x^5 \ln(x) \quad \int f_6(x)dx = \frac{x^6}{6} \ln(x) - \frac{x^6}{36} + C$

2. (30 points) There is a window in a dam and the center (incenter) of the window is 10 m below the surface of the water. Find the hydrostatic force upon the window in each of the following cases, or explain why you cannot find it in the particular instance.

- i The window is a square of area $25m^2$.
- ii The window is a circle of area $25m^2$.
- iii The window is a rectangle of area $25m^2$.
- iv The window is an isosceles triangle of area $25m^2$.
- v The window is an equilateral triangle of area $25m^2$.

- i The window is a square of area $25m^2$. So we have a square of length $5m$ on each side, and get the following when the axis is at the top of the water, centered on the figure, with positive y going up.

$$\int_{-12.5}^{-7.5} \rho g(-y)(5dy) = 250\rho g = 2,450,000N$$

** HOWEVER, this assumes the square is on the level, so to speak. No where does it say that, so in fact the square could be tilted, which will change the value.

- ii The window is a circle of area $25m^2$. Centering the axis in the middle of the circle, we have a circle of radius $5/\sqrt{\pi}$, so $x^2 + y^2 = 25/\pi$

$$\int_{-5/\sqrt{\pi}}^{5/\sqrt{\pi}} \rho g(10-y)(2x)dy = \int_{-5/\sqrt{\pi}}^{5/\sqrt{\pi}} \rho g(10-y)(2\sqrt{\frac{25}{\pi} - y^2})dy = 250\rho g = 2,450,000N$$

- iii The window is a rectangle of area $25m^2$. Since we have no idea of the dimensions of the rectangle, and hydrostatic force depends upon the depth of the window, we cannot do this problem.
- iv The window is an isosceles triangle of area $25m^2$. Similar to the rectangle, we have no idea of how wide the triangle is, or if the two congruent sides are both up are more level, we cannot do this one.
- v The window is an equilateral triangle of area $25m^2$. Same idea as the square, we do not know the orientation of the triangle, so the force would be different as you spin the triangle.

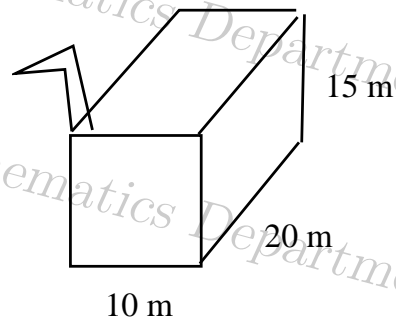
3. (30 points) Write each of the following into partial fractions of two fractions total. Be sure to make mention of any patterns you find.

I $\frac{1}{x(x-1)} = \frac{-1}{x} + \frac{1}{x-1}$

II $\frac{1}{x(x-2)} = \frac{-1/2}{x} + \frac{1/2}{x-1}$

III $\frac{1}{x(x-3)} = \frac{-1/3}{x} + \frac{1/3}{x-1}$

IV $\frac{1}{x(x-4)} = \frac{-1/4}{x} + \frac{1/4}{x-1}$



4. (30 points) We have a tank in the shape of a rectangle that is $10m$ by $20m$ and $15m$ high. The outlet spout is $8m$ above the top of the tank. For each of the integrals below (A-F), MATCH the description (I - VI) of where the axis was centered, and in which direction (up / down) is the positive y with the integral.

(A) $\int_0^{15} 1000(9.80)(y+8)(10)(20)dy$ (B) $\int_{-15}^0 1000(9.80)(8-y)(10)(20)dy$ (C) $\int_0^{15} 1000(9.80)(y+8)(10)(20)dy$

(D) $\int_{-8}^{-23} 1000(9.80)(-y)(10)(20)dy$ (E) $\int_{-2}^{13} 1000(9.80)(21-y)(10)(20)dy$ (F) $\int_{17}^{-2} 1000(9.80)(6-y)(10)(20)dy$

- I At the top of the pipe, up
- II $2m$ up from the bottom middle of the box, up
- III At the top middle of the box, down
- IV At the bottom middle of the box, up
- V $6m$ down from the top of the pipe, up
- VI At the top middle of the box, up

ANSWERS: A-IV, B-VI, C-III, D-I, E-II, F-V

5. (30 points) We have four different water tanks, each $42m$ long, and each has a spout $8m$ above the top of the tank through which we will pump out the water. However,

- (A) has an inverted isosceles triangle $10m$ wide and $10m$ high for its ends,
- (B) has an upright isosceles triangle $10m$ wide and $10m$ high for its ends,
- (C) has the lower half of a semi-circle of radius $10m$ for its ends, and
- (D) has the upper half of a semi-circle of radius $10m$ for its ends.

Match each of the eight integrals below with the correct face and location of the axis for determining the total work needed to pump the water out of the tank.

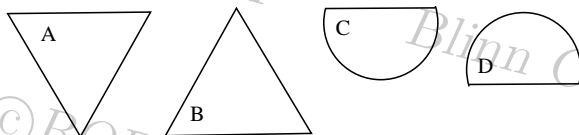
- (i) A, bottom, up (ii) B, bottom, up (iii) C, bottom, up (iv) D, bottom, up
- (v) A, top, up (vi) B, top, up (vii) C, top, up (viii) D, top, up

(I) $\int_0^{10} 1000(9.80)(18-y)(42)\left(2\frac{y}{2}\right)dy$ (II) $\int_0^{10} 1000(9.80)(18-y)(42)\left(2\frac{10-y}{2}\right)dy$

(III) $\int_0^{10} 1000(9.80)(18-y)(42)(2\sqrt{10^2-(y-10)^2})dy$ (IV) $\int_0^{10} 1000(9.80)(18-y)(42)(2\sqrt{10^2-y^2})dy$

(V) $\int_{-10}^0 1000(9.80)(8-y)(42)\left(2\frac{10+y}{2}\right)dy$ (VI) $\int_{-10}^0 1000(9.80)(8-y)(42)\left(2\frac{y}{2}\right)dy$

(VII) $\int_{-10}^0 1000(9.80)(8-y)(42)(2\sqrt{10^2-y^2})dy$ (VIII) $\int_{-10}^0 1000(9.80)(8-y)(42)(2\sqrt{10^2-(y+10)^2})dy$



ANSWERS: i-I, ii-II, iii-III, iv-IV, v-V, vi-VI