Math 2414 Take Home Exam One Questions, due at the end of the review day.

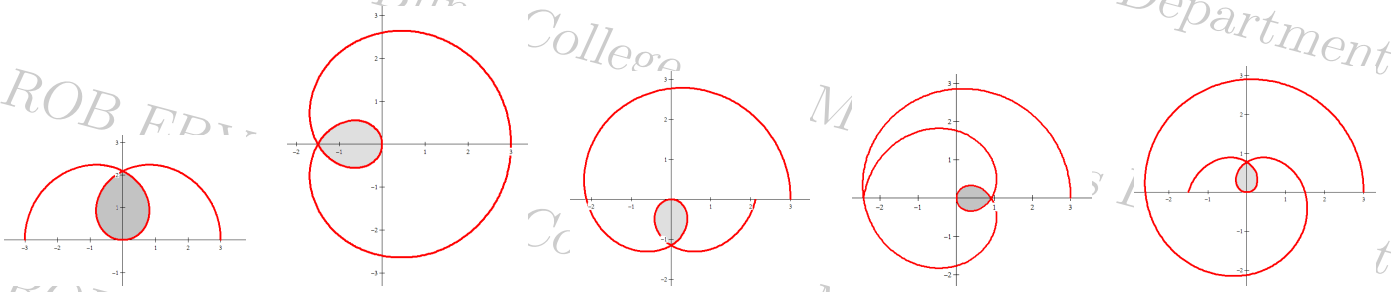
Group Members Names

1. Let $\boxplus(x)$ denote the greatest integer $\leq x, f(x) = \boxplus(x), g_2(x) = \boxplus(2x), g_3(x) = \boxplus(3x), g_4(x) = \boxplus(4x), h_2(x) = \boxplus(x/2), h_3(x) = \boxplus(x/3), h_4(x) = \boxplus(x/4)$. Find the following, and note what patterns you observe.

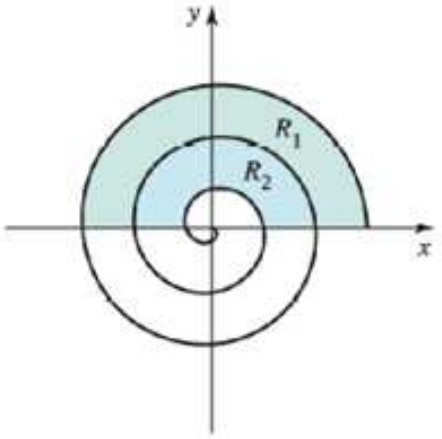
$$(1)\int_0^6 f(x)\,dx \quad (2)\int_0^6 g_2(x)\,dx \quad (3)\int_0^6 g_3(x)\,dx \quad (4)\int_0^6 g_4(x)\,dx \quad (5)\int_0^6 h_2(x)\,dx \quad (6)\int_0^6 h_3(x)\,dx \quad (7)\int_0^6 h_4(x)\,dx$$

2. Find the area of each *inner loop* (shaded) for each of the following polar curves. Make note of any patterns you see here. (All graphs except the first are on $[0, 4\pi]$, the first on $[0, 2\pi]$.

$$r = 3\cos(t/2) \quad r = 3\cos(t/3) \quad r = 3\cos(t/4) \quad r = 3\cos(t/5) \quad r = 3\cos(t/6)$$



3. Let R_n be the region bounded by the nth and the $(n+1)$st turn of the spiral $r = e^{-\theta}$ in the first and second quadrants, for $\theta \geq 0$ (See the figure)



A Set up the integral(s) to find the area A_n of R_n for $n = 1, 2, 3, 4$.

B Now evaluate each of the integrals you found above.

C Compute each of the following: A_1/A_2, A_2/A_3, A_3/A_4.

4. Average value is discussed in the text on page 379 in section 5.4. Arrange the following intervals in the order for which the average value of $\sin x$ over the interval increases from smallest to largest. (Hint: a is last in the list)

a $0 \leq x \leq \pi$

b $\pi/2 \leq x \leq 3\pi/2$

c $\pi \leq x \leq 2\pi$

d $3.14 \leq x \leq 3.15$

In addition to finding the average value numerically, you should also explain your answer like you would if this showed up on the in class exam. (So you should argue from the shape and features of the graph of $\sin x$)