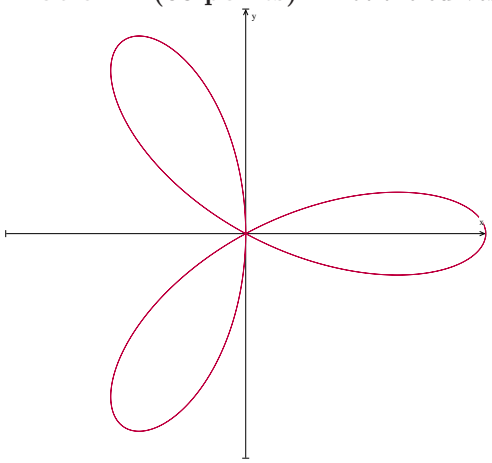
Math 2414 Quiz January 25, 2017 Unsupported answers earn NO credit

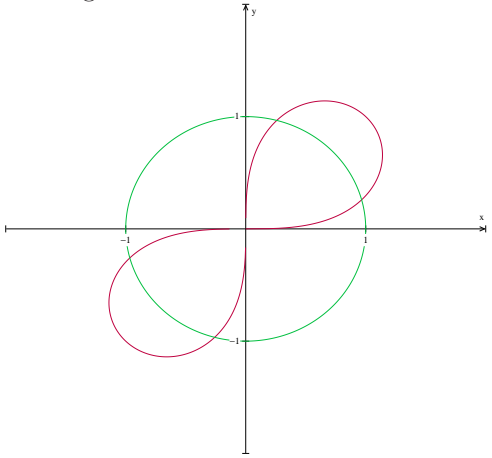
Problem 1 (10 points) Name and Section Number

Problem 2 (60 points) Plot the curves and find the area of the following regions: The region inside one leaf of $r = \cos 3\theta$



$$2\int_0^{\pi/6} \frac{1}{2}\left(\cos(3\theta)\right)^2\, d\theta = \int_0^{\pi/6} \left(\cos(3\theta)\right)^2\, d\theta = \int_0^{\pi/6} \frac{1+\cos(6\theta)}{2} d\theta = \left(\frac{1}{2}\theta + \frac{\sin(6\theta)}{12}\right)\Bigg|_0^{\pi/6} = \frac{\pi}{12} + \frac{0}{12} - 0 - 0 = \frac{\pi}{12}$$

The region inside the lemniscate $r^2 = 2\sin 2\theta$ and outside $r = 1$

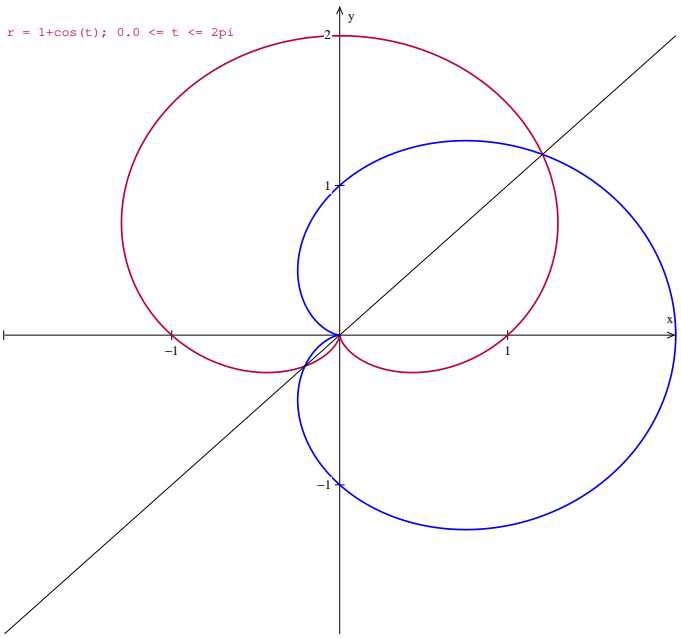


We need to first find the intersection points between the two functions. So we set them equal and solve $1^2 = 2\sin 2\theta$ to get two places, $\theta = \frac{\pi}{12}, \theta = \frac{5\pi}{12}$. Also, remember the function is r^2, so that helps with evaluating the integral.

$$2 \times \frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \left(2\sin(2\theta) - 1^2\right)\, d\theta = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \left(2\sin(2\theta) - 1\right)\, d\theta = \left(-\cos(2\theta) - \theta\right)\Big|_{\frac{\pi}{12}}^{\frac{5\pi}{12}} = \sqrt{3} - \frac{\pi}{3}$$

Problem 3 (30 points) Find the intersection points and then find the area that lies within both of the following pairs of curves.

$$r = 1 + \sin\theta \qquad r = 1 + \cos\theta$$



There are four parts to this area.
(A) The larger area bounded by the red $r = 1 + \sin\theta$ and the black line. This lies in the 4th and 1st quadrants.
(B) The larger area bounded by the blue $r = 1 + \cos\theta$ and the black line. This lies in the 1st and 2nd quadrants.
(C) The smaller area bounded by the blue $r = 1 + \cos\theta$ and the black line. This lies in the 3rd quadrant.
(D) The smaller area bounded by the red $r = 1 + \sin\theta$ and the black line. This lies in the 3rd quadrant.

$$(A) = \int_{\frac{-\pi}{2}}^{\frac{\pi}{4}} \frac{1}{2}(1+\sin\theta)^2 \, d\theta \qquad (B) = \int_{\frac{-\pi}{4}}^{\pi} \frac{1}{2}(1+\cos\theta)^2 \, d\theta \qquad (C) = \int_{\pi}^{\frac{5\pi}{4}} \frac{1}{2}(1+\cos\theta)^2 \, d\theta \qquad (D) = \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}} \frac{1}{2}(1+\sin\theta)^2 \, d\theta$$

$$\int \frac{1}{2}(1+\sin\theta)^2 \, d\theta = \int \frac{1+2\sin\theta+\sin^2\theta}{2} d\theta = \frac{6\theta - 8\cos\theta - \sin 2\theta}{8} + C$$

$$\int \frac{1}{2}(1+\cos\theta)^2 \, d\theta = \int \frac{1+2\cos\theta+\cos^2\theta}{2} d\theta = \frac{6\theta + 8\cos\theta - \sin 2\theta}{8} + C$$

$$(A) = \frac{-2-8\sqrt{2}+9\pi}{16} \qquad (B) = \frac{-2-8\sqrt{2}+9\pi}{16} \qquad (C) = \frac{2-8\sqrt{2}+3\pi}{16} \qquad (D) = \frac{2-8\sqrt{2}+3\pi}{16}$$

$$\text{Final Answer} \frac{-32\sqrt{2}+24\pi}{16} = -2\sqrt{2} + \frac{3\pi}{2}$$