Memo V Calculus 2

1. Suppose you know $\int\limits_1^\infty f_C\,dx$ converges and $\int\limits_1^\infty f_D\,dx$ diverges. Explain why the following either converge or diverge or if it is impossibe to tell: (I) $\int\limits_1^\infty f_C|\sin x|\,dx$ (II) $\int\limits_1^\infty f_C \arctan x\,dx$ (III) $\int\limits_1^\infty f_D|\sin x|\,dx$ (IV) $\int\limits_1^\infty f_D \arctan x\,dx$

2. Suppose you know $\int\limits_1^\infty f_C\,dx$ converges and $\int\limits_1^\infty f_D\,dx$ diverges. Explain why the following either converge or diverge or if it is impossibe to tell: (I) $\int\limits_1^\infty f_C(-e^{-x}+1)\,dx$ (II) $\int\limits_1^\infty f_C(e^{-x}+1)\,dx$ (III) $\int\limits_1^\infty f_D(-e^{-x}+1)\,dx$ (IV) $\int\limits_1^\infty f_D(e^{-x}+1)\,dx$

3. Let $\{r_n\}$ be the sequence given below. Evaluate $\lim\limits_{n\to\infty} r_n$ or show that it does not exist.

$$r_n = \begin{cases} n^2/(n^2-20) & \text{if } n \text{ is a multiple of } 3 \\ n/(n+1) & \text{if } n \text{ is a one more than a multiple of } 3 \\ \sqrt{n}/\sqrt{n+3} & \text{if } n \text{ is one less than a multiple of } 3 \end{cases}$$

4. Let $\{r_n\}$ be the sequence given below. Evaluate $\lim\limits_{n\to\infty} r_n$ or show that it does not exist.

$$r_n = \begin{cases} \frac{2n}{n-1} & \text{if } n \text{ is a multiple of } 3 \\ \frac{2n}{n+1} & \text{if } n \text{ is a one more than a multiple of } 3 \\ \frac{n^2}{n^2+10} & \text{if } n \text{ is one less than a multiple of } 3 \end{cases}$$

5. Show that $\frac{4^n}{n!}$ is eventually decreasing (starting at what term?), thefore has a limit (why?) and find the limit.

6. Professor Eby is considering a new grading scheme for next semester. Your grade would be based on the average of your exam scores, and you are allowed to take the exams as often as you like. If each of your exam scores is no worse than the average of your previous scores, show that your average score approaches a limit. (Each exam has a maximum score of 100%)

7. A series $\sum\limits_{k=1}^{\infty} r_k$ has partial sums s_n given by $s_n = 5 - 3/n$

   - Does $\sum\limits_{k=1}^{\infty} r_k$ converge, and if so to what?
   - Find $\lim\limits_{k\to\infty} r_k$
   - Find $\sum\limits_{k=1}^{100} r_k$

8. If $\sum a_k$ converges and $a_n \geq 0$, show / explain why $\sum \sin(a_k)$ converges

9. Explain, for a College Algebra student, why $\frac{3n+4}{2n^2+3n+5} \geq \frac{3}{10n}, n \geq 1$

10. Consider $f(x) = 1/x$ and take the n Right hand Riemann sums over $[1, n]$. What series is this starting to form, what integral for the integral test would this relate to, and does it show convergence or divergence? NOW consider $f(x) = 1/x$ and take the n Left hand Riemann sums over $[1, n]$. What series is this starting to form, what integral for the integral test would this relate to, and does it show convergence or divergence?

11. For each ofthe following situations, decide if $\sum_{n=1}^{\infty} c_n$ converges, diverges, or if you cannot determine. Then, for each one where you need more information, give an example of a series that converges and an example of a series that diverges.

a $0 \leq c_n \leq \frac{1}{n}$ for all n
b $\frac{1}{n} \leq c_n$ for all n
c $0 \leq c_n \leq \frac{1}{n^2}$ for all n
d $\frac{1}{n^2} \leq c_n$ for all n

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