Memo IV Calculus 2

1. Explain how the following two integrals are equivelent, show the steps to get one to the other, what Trig substitution is needed in each case, and explain why it works.

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx \qquad -\int \sqrt{1-r^2}\,dr$$

2. Define a curve as given below.
   - Give a graph over $[0, 8\pi]$
   - Find the equation of the tanget line at $(1, 0)$
   - Find the arc length over $[0, 8\pi]$
   - Set up the integral for the area in the first quadrant bounded on the outside by this curve over $[0, \pi/2]$. Explain why it is 'obvious' that $\pi/4$ is an upper bound for this integral.

   Each line of mathematics here counts as 10 words, there must be at least 4 lines in the computations.

$$x = e^{-0.1t}\cos t \qquad y = e^{-0.1t}\sin t$$

3. A window is cut into the vertical side of a deep swimming pool. This window is rectangular with an area of $1m^2$, and th ecenter is exactly $3m$ beneath the surface of the water. Compute the force of water on the window, or explain why you cannot. What shape window could you calculate the force for?

4. Explain in words what each of the following is computing. Also find the value of each integral. Each line of mathematics here counts as 10 words, there must be at least 4 lines in the computations. HINT: at least one of these is finding the length of the curve $y = \sqrt{x}$ from a to b.

$$\int_a^b \sqrt{x^2+x}\,dx \qquad \int_a^b \sqrt{1+\frac{1}{2\sqrt{x}}}\,dx \qquad \int_a^b \sqrt{1+\frac{1}{4x}}$$

5. For each of the following functions find the value of R for which the line $x = R$ divides the area of the region under the graph from $x = a$ to $x = b$ in a 2:1 ratio.

$$f_1(x) = (x-2)^2 \quad a = -1, b = 1 \qquad f_2(x) = 1 - 1/x^2 \quad a = 1, b = 2$$

6. There are three windows, each of a different size, (vertical height x horizontal width) 1mx4m, 2mx2m, 4mx1m windows and each can stand 90,000 N. How deep can the tank be filled, measured from the top of each window?