Memo III Calculus 2

1. We are constructing a figure whose base is the square with verticies at $(0,0),(0,\pi),(\pi,0),(\pi,\pi)$ and the height of each vertical strip is given by $\sin x$. What is the resulting volume, and to what average value calculation does this naturally relate? Each line of mathematics here counts as 10 words, there must be at least 4 lines in the computations.

2. We are constructing a figure whose base is the square with verticies at $(0,0),(0,\pi),(\pi,0),(\pi,\pi)$ and the height of each vertical strip is given by $\cos x$. What is the resulting volume, and to what average value calculation does this naturally relate? Each line of mathematics here counts as 10 words, there must be at least 4 lines in the computations.

3. Consider the region bounded below by $y_1 = x^m$ and above by $y_2 = x^n$, $0 \le x \le 1$ and $0 \le n < m$. Find the general formula depending on n and m for the x and y coordinate of the center of mass of this region of constant density. Can you find n and m so that the center is OUTSIDE the region? Explain how you found each answer.

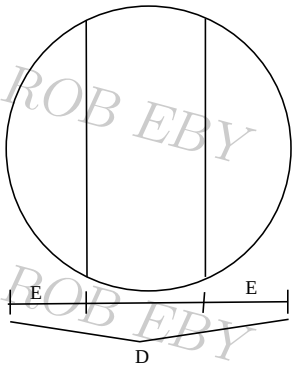
4. Evaluate $\int \tan x \sec^2 x \, dx$ with $u = \tan x$ and then with $u = \sec x$. Explain why you get 'different' answers.

5. Evaluate $\int \cot x \csc^2 x \, dx$ with $u = \cot x$ and then with $u = \csc x$. Explain why you get 'different' answers.

6. Evaluate $\int \cos x \sin x \, dx$ with $u = \cos x$ and then with $u = \sin x$. Explain why you get 'different' answers.

7. Consider the region bounded below by $y_1 = m \ln x$ and above by $y_2 = n \ln x$, $1 \le x \le 2$ and $0 < m < n$. Find the general formula depending on n and m for the x and y coordinate of the center of mass of this region of constant density. Can you find n and m so that the center is OUTSIDE the region? Explain how you found each answer.

8. Suppose you and two friends go split a pizza, and cut two vertical lines as shown below. What value(s) of E relative to D give equal areas? What values of E give equal amounts of crust? Explain why in general these will NOT be the same value, and find (or explain why it cannot happen) the diameter and cut values that do give both equal areas and crusts.



9. Explain two ways, via geometry and with calculus, which of these will be larger.

$$\int_0^R x\sqrt{R^2 - x^2}\, dx \qquad \int_0^R \sqrt{R^2 - x^2}\, dx$$

10. You and three friends split a loaf of bread that just happens to be shaped exactly like a hemisphere. If you each want an equal amount of crust (no crust on the circle side) and you want to make the cuts symmetric, where do the cuts need to be made? (So a top view is given below.