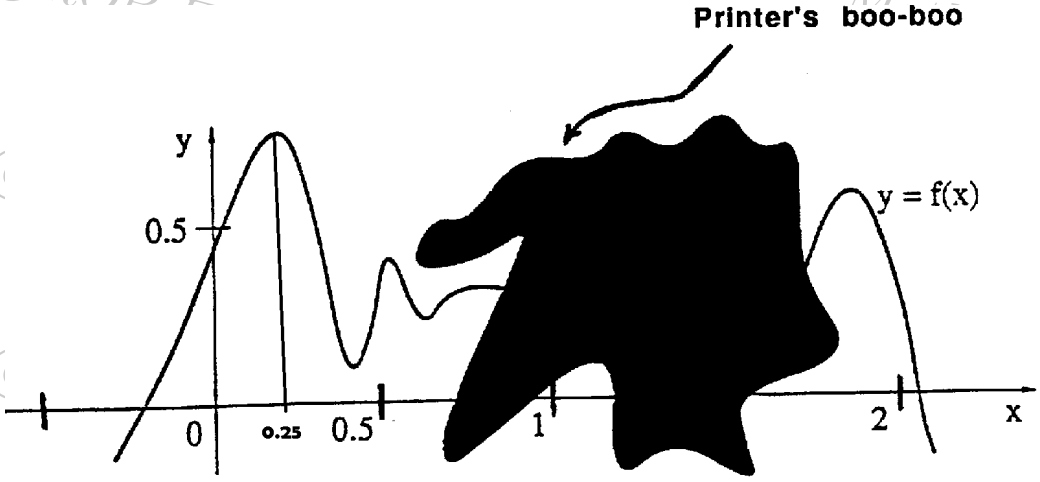
Memo I Calculus 2

1. Suppose R is an integrable function and that $\int_0^1 R(x)dx = 2, \int_0^2 R(x)dx = 1, \int_2^4 R(x)dx = 7$.
   (a) Find and explain why $\int_1^2 R(x)dx$
   (b) Find and explain why $\int_1^4 R(x)dx$
   (c) Explain why $R(x) \geq 3.5$ for at least one value of $x \in [2, 4]$.

2. Consider the function graphed on $[0, 2]$ below with the *printers error*. Explain why each of the following is positive, negative, exactly zero, or undetermined. $\int_0^2 f''(x)dx \quad \int_0^2 f'(x)dx \quad \int_0^2 f(x)dx$

3. Consider the function graphed on $[0, 2]$ below with the *printers error*. Explain why each of the following is positive, negative, exactly zero, or undetermined. $\int_{0.25}^2 f''(x)dx \quad \int_{0.25}^1 f'(x)dx \quad \int_{0.25}^1 f(x)dx$

4. Consider the function graphed on $[0, 2]$ below with the *printers error*. Explain why each of the following is positive, negative, exactly zero, or undetermined. $\int_0^{0.25} f''(x)dx \quad \int_1^2 f'(x)dx \quad \int_1^2 f(x)dx$



5. Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x), x \in \Re$, then explain why each of the following MUST, MIGHT, or are NEVER true:
   - $f'(x) \leq g'(x), x \in \Re$
   - $f''(x) \leq g''(x), x \in \Re$
   - $\int_a^b f(x)dx \leq \int_a^b g(x)dx$

6. If f is continuous and (strictly) increasing on the interval $[a, b]$ with $f(a) > 0$, then explain why each of the following MUST, MIGHT, or are NEVER true:
   - $\int_a^b f(x)dx < f(b)(b-a)$
   - $\int_a^b f(x)dx > f(a)(b-a)$
   - $\int_a^b f(x)dx = f(c)(b-a)$ for some $c \in (a, b)$

7. Let $F(t) = \int_0^3 t\sqrt{t+9}\,dt$ Explain why $F'(1) = 0$ Now let $R(t) = \int_0^3 t\sqrt{t+9}\,dt$ Explain what $R'(1)$ must be, and what the difference is between $F(t)$ and $R(t)$.

8. Consider the triangle whose vertices in the rectangular coordinate system are $(0,0), (1,0), (1,1)$. (I) Find polar coordinate equations for each of the lines in which a side of the triangle lies (II) Set up (and explain and solve) a polar integral to show the area of this triangle is indeed $1/2$.

9. Explain, to a Calculus I student, the difference between the two solution methods: $\int \frac{-2e^{2x}}{1+e^{4x}}dx$ and $\int \frac{-2e^{4x}}{1+e^{4x}}dx$

10. Explain, to a Calculus I student, the difference between the two solution methods: $\int \frac{-2e^{-2x}}{1+e^{-4x}}dx$ and $\int \frac{-2e^{-4x}}{1+e^{-4x}}dx$

11. (r1) Explain two ways, with AND without Calculus. Use the graph below to answer: What is the area between the two curves from $-R$ to R if

    - f is odd and g is even?
    - f is even and g is odd?
    - Both f and g are odd?
    - Both are even?