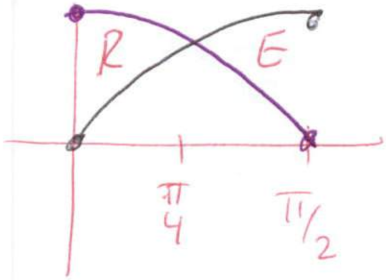
Math 2414 Quiz January 19, 2017 Unsupported answers earn NO credit

**Problem 1 (10 points)** Name and Section Number

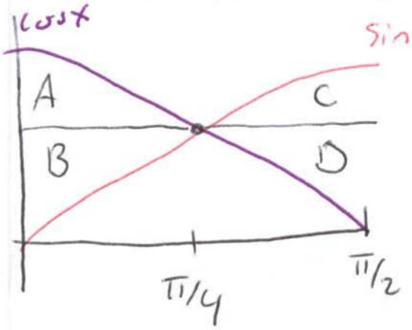
**Problem 2 (20 points)** SET UP the integral(s) required to find the area between $y = \sin x, y = \cos x, y = 0$ between $x = 0, x = \pi/2$ with rectangels on the $x$-axis.




"R" "E"

$$\int_0^{\pi/4} \left(\cos(x) - \sin(x)\right) dx + \int_{\pi/4}^{\pi/2} \left(\sin(x) - \cos(x)\right) dx$$

**Problem 3 (20 points)** SET UP the integral(s) required to find the area between $y = \sin x, y = \cos x, y = 0$ between $x = 0, x = \pi/2$ with rectangels on the $y$-axis.

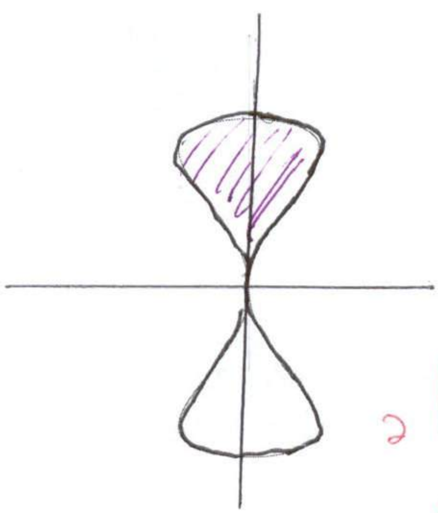



A B C D

$$\int_{1/\sqrt{2}}^{1} \cos^{-1}(y)\, dy + \int_0^{1/\sqrt{2}} \sin^{-1}(y)\, dy + \int_{1/\sqrt{2}}^{1} \left(\frac{\pi}{2} - \sin^{-1}(y)\right) dy + \int_0^{1/\sqrt{2}} \left(\frac{\pi}{2} - \cos^{-1}(y)\right) dy$$

**Problem 4 (40 points)** Determine the area of the region inside the curve and above the horizontal axis for the curve given below:

$$x^2 = y^4(1 - y^3)$$



on x-axis

$$\int_0^{\bigcirc} \text{ugly!}$$

on y-axis

$$2\int_0^1 \sqrt{y^4(1-y^3)}\, dy$$

$$2\int_0^1 y^2\sqrt{1-y^3}\, dy \qquad u = 1 - y^3 \qquad \frac{du}{dy} = -3y^2 \qquad 2\int_1^0 u^{1/2} \frac{du}{-3}$$

$$\frac{2}{3}\int_0^1 u^{1/2}\, du = \frac{2}{3} \left.\frac{u^{3/2}}{3/2}\right|_0^1 = \left.\frac{4}{9} u^{3/2}\right|_0^1 = \frac{4}{9} - 0 = \frac{4}{9}$$

**Problem 5 (20 points)** Eliminate the parameter to obtain an equation in $x$ and $y$. Describe the curve and indicate the positive orientation, and give the starting and ending point of the curve.

$$x = \cos t \qquad y = \sin^2 t \qquad 0 \leq t \leq \pi$$

Remember,

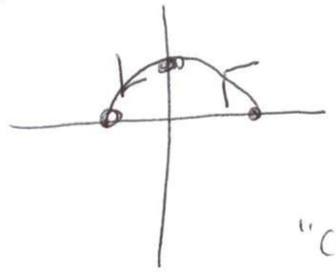
$\sin^2\theta + \cos^2\theta = 1$

$y + x^2 = 1$

or

$y = 1 - x^2$

| $t$ | $x$ | $y$ |
|---|---|---|
| 0 | 1 | 0 |
| $\pi/2$ | 0 | 1 |
| $\pi$ | $-1$ | 0 |

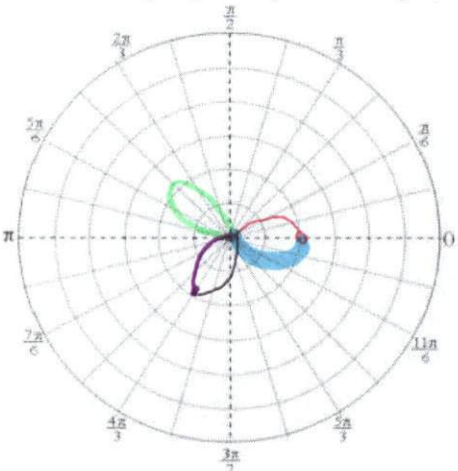


Starts at $(1,0)$

ends $(-1,0)$

goes "Counter Clockwise"

**Problem 6 (20 points)** Draw a graph of $r = \cos 3\theta$. The entire radius of the graph is three, like in the notes.




| $\theta$ | $r$ |
|---|---|
| 0 | 1 |
| $\pi/6$ | 0 |
| $\pi/4$ | |
| $\pi/3$ | |
| $\pi/2$ | 0 |

So a "rose" with three loops, or "petals"

**Problem 7 (20 points)** Convert the following functions into polar:

(A) $y = 3$ $\quad 3 = r\sin\theta \qquad r = 3\csc\theta$

(B) $y = x^2$ $\quad r\sin\theta = r^2\cos^2\theta \longrightarrow r = \dfrac{\sin\theta}{\cos^2\theta} \Rightarrow r = \tan\theta\sec\theta$

(C) $(x-1)^2 + y^2 = 1$ $\quad r = 2\cos\theta$

$x^2 - 2x + 1 + y^2 = 1$

$r^2 - 2r\cos\theta = 0 \rightarrow r = 2\cos\theta$

(D) $y = 1/x$

$r\sin\theta = \dfrac{1}{r\cos\theta}$

$r^2 \sin\theta\cos\theta = 1$