Math 2414 Quiz March 2, 2017 Unsupported answers earn NO credit

Problem 1 (10 points) Name and Section Number

Problem 2 (60 points) What is the appropriate substitution for each of the following integrals? (If none (i.e. it is a basic rule) , state why)

$\int 5x^4\,dx$ none, basic rule

$\int x\sqrt{9-x^2}\,dx$ $u = 9 - x^2$ $du = -2x\,dx$

$\int \frac{e^{2t}}{1+e^{2t}}$ $u = 1 + e^{2t}$ $\frac{du}{dt} = 2e^{2t}$

$\int \frac{dx}{\sqrt{64-4x^2}} = \int \frac{dx}{\sqrt{4(16-x^2)}} = \frac{1}{2}\int \frac{dx}{\sqrt{16-x^2}}$

trig sub $x = 4\sin\theta$ 4 x $\sqrt{4^2-x^2}$

$\int \frac{r\,dr}{\sqrt{r^4-81}} = \int \frac{r\,dr}{\sqrt{(r^2)^2-9^2}}$

trig sub

$r^2 = 9\sec\theta$

$\int \frac{dt}{1+e^{2t}}$

$\tan\theta = e^t$ also has problems

$u = e^{2t} \to \int \frac{du}{2u(1+u)}$

$\int \frac{e^t\,dt}{1+e^{2t}}$ would be $\tan^{-1}(e^t) + c$

Problem 3 (30 points) What is the best first step in evaluating the following integrals?

$\int \sin^3 x \cos^2 x\,dx = \int \sin x(1-\cos^2)\cos^2 x\,dx = \int \sin x\cos^2 x\,dx - \int \sin x\cos^4 x\,dx$

$\int \sin^2 x \cos^2 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)dx$

Problem 4 (50 points) Evaluate the following:

$\int \frac{\sin^3 x}{\cos^5 x}\,dx = \int \frac{\sin x(1-\cos^2 x)}{\cos^5 x}\,dx = \int \frac{\sin x}{\cos^5 x}\,dx - \int \frac{\sin x}{\cos^3 x}\,dx$

$= \int \sin x\cos^{-5} x\,dx + \int \sin x\cos^{-3} x\,dx$

$u = \cos x$

$du = -\sin x\,dx$

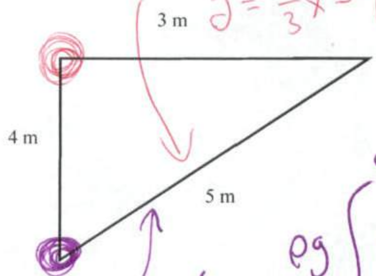
$= (-1)\frac{(\cos x)^{-4}}{-4} - \frac{(-1)(\cos x)^{-2}}{-2} + C$

$\frac{1}{4(\cos x)^4} + \frac{1}{2(\cos x)^2} + C$

OR $\int \tan^3 x\sec^2 x\,dx$

$u = \tan x$ $\boxed{\frac{\tan^4 x}{4} + C}$

Problem 5 (30 points) SET UP the integral(s) to find the hydrostatic force on the following dam. Assume the water is at the top of the triangle. Indicate with a dot where you are setting the origin of the axis.



$y = \frac{4}{3}x - 4$

$\rho g \int_{-4}^{0} (-y)\, x \, dy = (9.80)(1000) \int_{-4}^{0} (-y)\left(\frac{3(y+4)}{4}\right) dy$

$y = \frac{4}{3}x$

$\rho g \int_{0}^{4} (4-y)(x)\, dy = 1000(9.80) \int_{0}^{4} (4-y)\, \frac{3y}{4}\, dy$

Problem 6 (30 points) Describe what your first step should be for the following problem: Find the constants E, B, Y so that the following works.

$$\frac{E}{x-3} + \frac{B}{x+4} + \frac{Y}{x-1} = \frac{x^3}{(x-1)(x+4)(x-3)}$$

$(x^2+3x-4)(x-3)$

$\frac{x^3}{x^3 - 13x + 12}$

$\boxed{\text{Polynomial Division}}$

$x^3 + 0x^2 - 13x + 12 \overline{\big) x^3 + 0x^2 + 0x + 0}$ 1

$-(x^3 + 0x^2 - 13x + 12)$

$13x - 12$

$x + \frac{13x - 12}{(x-1)(x+4)(x-3)}$

Problem 6 (50 points) Evaluate the following:

$$\int \frac{81}{x^3 - 9x^2} dx$$

$81 \int \frac{1}{x^2(x-9)} dx = 81 \left(\int \left(\frac{-\frac{1}{81}}{x} + \frac{-\frac{1}{9}}{x^2} + \frac{\frac{1}{81}}{x-9} \right) dx = \int \left(\frac{-1}{x} + \frac{-9}{x^2} + \frac{-1}{x-9} \right) dx \right.$

$\boxed{-\ln|x| + \frac{9}{x} - \ln|x-9| + C}$

$\frac{E}{x} + \frac{B}{x^2} + \frac{Y}{x-9} = \frac{1}{x^2(x-9)}$

$Ex(x-9) + B(x-9) + Yx^2 = 1$

$x=0 \quad -9B = 1 \quad B = -1/9$

$x=9 \quad 81Y = 1 \quad Y = \frac{1}{81}$

$x=1 \quad -8E - 8B + Y = 1$

$-8E + \frac{8}{9} + \frac{1}{81} = 1$

$-8E = \frac{81}{81} - \frac{1}{81} - \frac{72}{81}$

$E = -1/81$