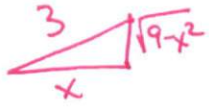
Math 2414 Quiz February 28, 2017 Unsupported answers earn NO credit

Problem 1 (10 points) Name and Section Number

Problem 2 (30 points) Evaluate the following:

$$\int \frac{dx}{\sqrt{18-2x^2}}$$

$$\int \frac{dx}{\sqrt{2}\sqrt{9-x^2}}$$



$x = 3\cos\theta$

$dx = -3\sin\theta\, d\theta$

$$\int \frac{-3\sin\theta\, d\theta}{\sqrt{2}\sqrt{9-9\cos^2\theta}}$$

$$\int \frac{-3\sin\theta\, d\theta}{\sqrt{2}\sqrt{9}\sqrt{1-\cos^2\theta}}$$

$$-\frac{1}{\sqrt{2}}\int \frac{\sin\theta\, d\theta}{\sin\theta}$$

$$-\frac{1}{\sqrt{2}}\int d\theta$$

$$-\frac{1}{\sqrt{2}}\theta + C$$

$$-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{9-x^2}}{x}\right) + C$$

Problem 3 (30 points) Mark the error in the following steps and fix it. If there is no error, state that.

$$\int \sin^3 x\cos^2 x\, dx = \int (\sin x)(\sin^2 x)(\cos^2 x)\, dx = \int (\sin x)\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$$

NO

$$\int \sin x\,(1-\cos^2 x)\cos^2 x\, dx = \int \sin x\cos^2 x - \int \sin x\cos^4 x\, dx$$

$$\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + C$$

$$\int \sin^2 x\cos^4 x\, dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2 dx = \int \left(\frac{1-\cos x-\cos^2 x+\cos^3 x}{8}\right) dx$$

ok

but

$$(1-\cos 2x)(1+2\cos 2x+\cos^2 2x) = 1+\cos 2x-\cos^2 2x-\cos^3 2x$$

Problem 4 (30 points) Evaluate the following:
HINT: I gave you enough room if you write normal size.....

$$\int \sin^2(x)\sec(x)\, dx$$

$$\int \frac{1-\cos^2 x}{\cos x}\, dx$$

$$\int \sin^2 x \frac{1}{\cos x}\, dx$$

$$\int \frac{\sin^2 x}{\cos x}\, dx$$

$$\int \frac{1}{\cos x}\, dx - \int \cos x\, dx$$

$$\int \sec x\, dx - \int \cos x\, dx$$

$$\ln\left|\sec x + \tan x\right| - \sin(x) + C$$

Problem 5 (80 points) Let R be the region given below.

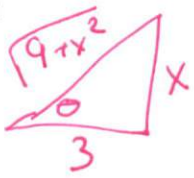
$$y = 0, x = 0, f(x) = (9+x^2)^{-1/2} \quad x \in [0,4]$$

(I) Find the area of R

$$\int_0^4 \frac{1}{\sqrt{9+x^2}}\,dx$$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$



$$\int_0^4 \frac{3\sec^2\theta\,d\theta}{\sqrt{9+9\tan^2\theta}}$$

$$\int_0^4 \sec\theta\,d\theta$$

$$\ln\left|\sec x + \tan x\right|\Big|_0^4$$

$$\ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right|\Big|_0^4$$

$$\ln\left|\frac{5}{3}+\frac{4}{3}\right| - \ln|1|$$

$\ln(3)$ units^2

(II) Find the volume generated when R is revolved about the x-axis. SET UP for both shells and washers, evaluate one

$$2\pi\int_{1/3}^{1/5} y\,x\,dy$$

$$2\pi\int_{1/3}^{1/5} y\sqrt{\frac{1}{y^2}-9}\,dy$$

$$\pi\int_0^4 (y)^2\,dx = \pi\int_0^4 \frac{1}{9+x^2}\,dx$$

$$\frac{\pi}{3}\tan\left(\frac{x}{3}\right)\Big|_0^4$$

$$\frac{\pi}{3}\tan^{-1}\left(\frac{4}{3}\right) - \frac{\pi}{3}\tan^{-1}(0)$$

$$\frac{\pi}{3}\tan^{-1}\left(\frac{4}{3}\right) \text{ units}^3$$

(III) Find the volume generated when R is revolved about the y-axis. SET UP for both shells and washers, evaluate one

$$2\pi\int_0^4 x\,y\,dx$$

$$2\pi\int_0^4 x\frac{1}{\sqrt{9+x^2}}\,dx$$

$u = 9+x^2$

$du = 2x\,dx$

$$2\pi\int_9^{25} \frac{1}{\sqrt{u}}\,\frac{du}{2}$$

$$\pi\int_9^{25} u^{-1/2}\,du$$

$$2\pi\sqrt{u}\Big|_9^{25}$$

$$2\pi\sqrt{25} - 2\pi\sqrt{9}$$

$$10\pi - 6\pi$$

4π units^3

$$\pi\int_{1/3}^{1/5} (x)^2\,dy$$

$$\pi\int_{1/3}^{1/5} \left(\sqrt{\frac{1}{y^2}-9}\right)^2 dy$$

$y = \frac{1}{\sqrt{9+x^2}}$

$y^2 = \frac{1}{9+x^2}$

$9+x^2 = \frac{1}{y^2}$

$x = \sqrt{\frac{1}{y^2}-9}$