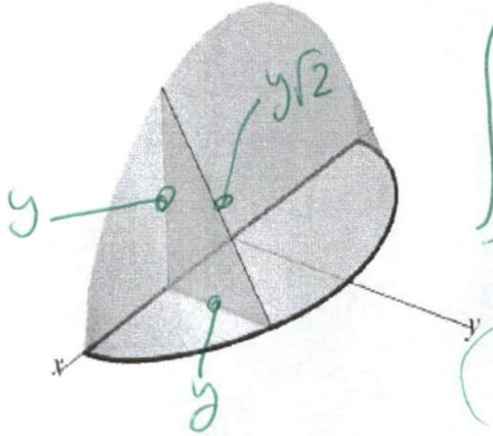
Math 2414 Quiz February 7, 2017 Unsupported answers earn NO credit

**Problem 1 (10 points)** Name and Section Number

**Problem 2 (40 points)** Set up the integral(s) and then FIND the volume of the solid whose base is $y = \sqrt{\cos x}$, $x \in [-\pi/2. \pi/2]$ and whose cross sections through the solid perpendicular to the $x$-axis are isosceles right triangles with a horizontal leg in the $xy$-plane and a vertical leg above the $x$-axis.




$$\int_{-\pi/2}^{\pi/2} \underbrace{\frac{1}{2} y^2}_{\text{Vol. of slab } \frac{1}{2}y^2} \, dx = \int_{-\frac{\pi}{2}}^{\pi/2} \frac{1}{2}\left(\sqrt{\cos x}\right)^2 dx = \int_{-\frac{\pi}{2}}^{\pi/2} \frac{\cos x}{2} \, dx$$

$$\frac{\sin x}{2}\Big|_{-\pi/2}^{\pi/2} = \frac{1}{2} - \left(-\frac{1}{2}\right) = \underline{1}$$

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$y = x - x^2$

$x^2 - x = -y$

Complete square!

$x^2 - x + \frac{1}{4} = -y + \frac{1}{4}$

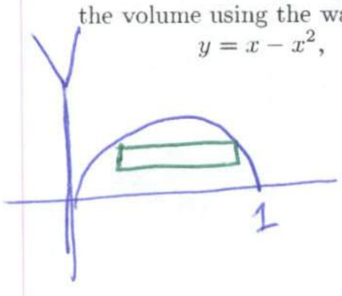
$\left(x - \frac{1}{2}\right)^2 = \frac{1}{4} - y$

$x - \frac{1}{2} = \pm\sqrt{\frac{1}{4} - y}$

$x = \frac{1}{2} \pm \sqrt{\frac{1}{4} - y}$

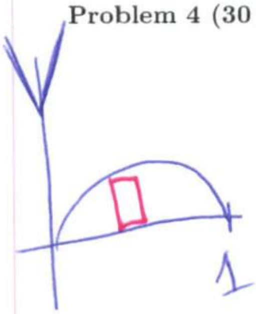
**Problem 3 (30 points)** Let R be the region bounded by the following curves. SET UP the integral(s) required to find the volume using the washer method when R is revolved around the $y$-axis.

$y = x - x^2, \quad y = 0$



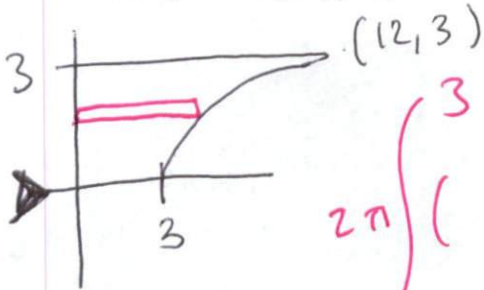

$$\pi \int_0^{1/4} \left( \left(\frac{1}{2} + \sqrt{\frac{1}{4} - y}\right)^2 - \left(\frac{1}{2} - \sqrt{\frac{1}{4} - y}\right)^2 \right) dy$$

**Problem 4 (30 points)** Now take the same set up as in problem 3, but SET UP the integral(s) for the shell method.



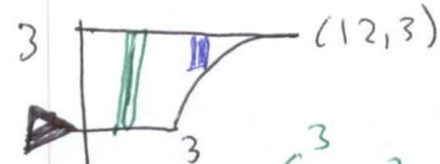

$$2\pi \int_0^1 (x)(y) \, dx = 2\pi \int_0^1 x(x - x^2) \, dx$$

**Problem 5 (30 points)** Let R be the region in the first quadrant bounded by the graph of $y = \sqrt{x-3}$ and the line $y = 3$. Set up the integral(s) required for the SHELL method to find the volume when R is revolved about the $x$-axis.




$$2\pi\int_0^3 (y)(x)\,dy = 2\pi\int_0^3 y(y^2+3)\,dy$$

**Problem 6 (30 points)** Now set up the integral(s) for the volume in number 5 but using the DISK / WASHER method.




$$\pi\int_0^3 (3)^2\,dx + \pi\int_3^{12}\left((3)^2-(\sqrt{x-3})^2\right)dx$$

**Problem 7 (30 points)** Pick one of the set ups above and evaluate it.

$$2\pi\int_0^3 (y^3+3y)\,dy = 2\pi\left(\frac{y^4}{4}+\frac{3y^2}{2}\right)\Big|_0^3$$

$$2\pi\left(\frac{3^4}{4}+\frac{3(3)^2}{2}\right) - 2\pi(0)$$

$$2\pi\left(\frac{81}{4}+\frac{27}{2}\right) = 2\pi\left(\frac{81+54}{4}\right) = \boxed{\frac{135\pi}{2}\text{ units}^3}$$