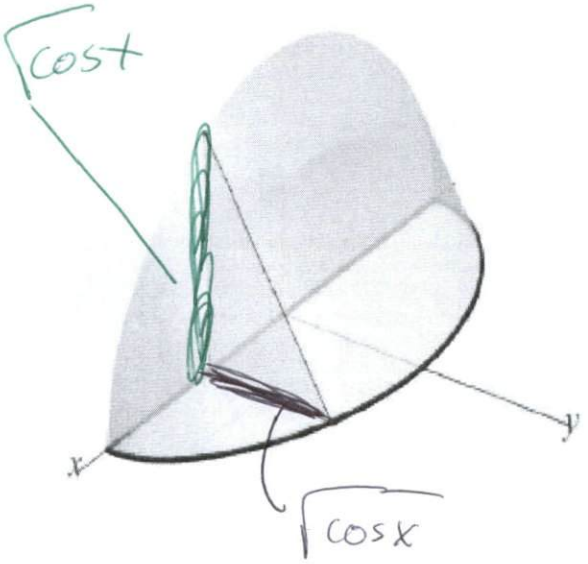
Math 2414 Quiz July 10, 2017 Unsupported answers earn NO credit

Problem 1 (10 points) Name and Section Number

Problem 2 (30 points) Find the volume of the solid whose base is $y = \sqrt{\cos x}$, $x \in [-\pi/2, \pi/2]$ and whose cross sections through the solid perpendicular to the x-axis are isosceles right triangles with a horizontal leg in the xy-plane and a vertical leg above the x-axis.



$$\int_{-\pi/2}^{\pi/2} \frac{1}{2}\left(\sqrt{\cos x}\right)^2 dx = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \cos(x)\, dx$$

$$\frac{1}{2}\sin(x)\Big|_{-\pi/2}^{\pi/2} = \frac{1}{2}(1) - \frac{1}{2}(-1) = \underline{\underline{1}}$$

Problem 3 (30 points) Let R be the region bounded by the following curves. SET UP the integral(s) required to find the volume using the washer method when R is revolved around the y-axis.

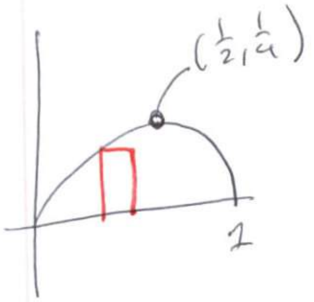
$y = x - x^2$, $y = 0$

Complete square to find

$x = \frac{1}{2} \pm \sqrt{1 - \frac{y}{4}}$

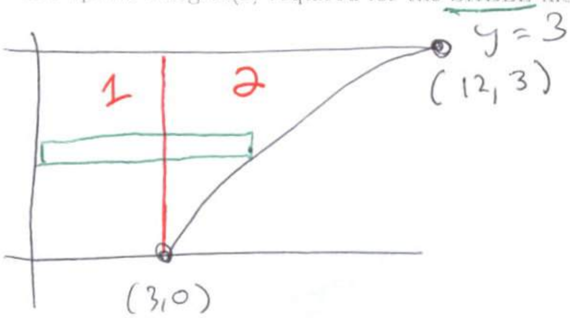
$$\int_0^{\frac{1}{4}} \left(\pi\left(\frac{1}{2} + \sqrt{1 - y/4}\right)^2 - \pi\left(\frac{1}{2} - \sqrt{1 - \frac{y}{4}}\right)^2\right) dy$$

Problem 4 (30 points) Now take the same set up as in problem 3, but set up the integral(s) for the shell method.



$$\int_0^1 2\pi(x)(x - x^2)\, dx$$

Problem 5 (30 points) Let R be the region in the first quadrant bounded by the graph of $y = \sqrt{x-3}$ and the line $y = 3$. Set up the integral(s) required for the SHELL method to find the volume when R is revolved about the x-axis.



$$\int_0^3 2\pi(y)\,x\,dy = \int_0^3 2\pi y(y^2+3)\,dy$$

Problem 6 (30 points) Now set up the integral(s) for the volume in number 5 but using the DISK / WASHER method.

1

$$\int_0^3 \pi(3-0)^2\,dx + \int_3^{12} \left[\pi(3)^2 - \pi\left(3-\sqrt{x-3}\right)^2\right] dx$$

2

Problem 7 (30 points) Pick one of the set ups above and evaluate it.

$$2\pi\int_0^3 (y^3+3y)\,dy = 2\pi\left(\frac{y^4}{4}+\frac{3y^2}{2}\right)\Bigg|_0^3$$

$$2\pi\left(\frac{3^4}{4}+\frac{3(3)^2}{2}\right) - 0$$

$$2\pi\left(\frac{81}{4}+\frac{27}{2}\right) = \pi\left(\frac{81}{2}+27\right) = \boxed{\pi\,\frac{135}{2}}$$