Math 2414 Quiz July 10, 2017 Unsupported answers earn NO credit

Problem 1 (10 points) Name and Section Number

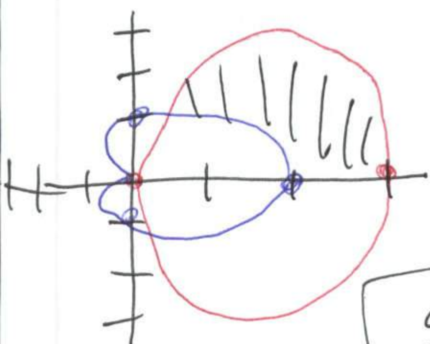
Problem 2 (20 points) Determine dy/dx in terms of t and evaluate it for the given value of t.

$$x = t + 1/t \quad y = t - 1/t; \qquad t = 1$$

$$\frac{\partial y}{\partial t} = 1 + \frac{1}{t^2} \qquad \frac{\partial x}{\partial t} = 1 - \frac{1}{t^2}$$

$$\frac{\partial y}{\partial x} = \frac{1 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = \frac{1 + \frac{1}{1}}{1 - \frac{1}{1}} = \frac{2}{0} = \frac{2}{0} \text{ undefined}$$

Problem 3 (50 points) Set up the integral(s) required, and then find the area that lies outside $r = 1 + \cos(\theta)$ but inside $r = 3\cos(\theta)$



$$1 + \cos\theta = 3\cos\theta$$

$$1 = 2\cos\theta$$

$$\frac{1}{2} = \cos\theta$$

$$\theta = \pi/3,\ 5\pi/3$$

$$2 \int_0^{\pi/3} \frac{1}{2}\pi \left[(3\cos\theta)^2 - (1 + \cos\theta)^2 \right] d\theta$$

double shaded area

Problem 4 (50 points) Find the slope of the tangent line to the following polar curve at the indicated point: $r = 1 - \sin(\theta) \quad (1/2, \pi/6)$

$$\frac{dy}{dx} = \frac{f'(\theta_0)\sin(\theta_0) + f(\theta_0)\cos(\theta_0)}{f'(\theta_0)\cos(\theta_0) - f(\theta_0)\sin(\theta_0)}$$

$$\frac{\partial y}{\partial x} = \frac{-\cos(\theta_0)\sin(\theta_0) + (1 - \sin(\theta_0))\cos(\theta_0)}{-\cos(\theta_0)\cos(\theta_0) - (1 - \sin(\theta_0))\sin(\theta_0)}$$

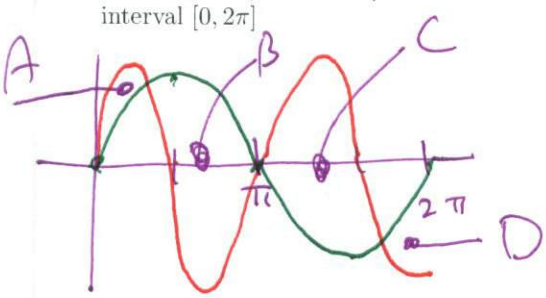
$$\frac{\partial y}{\partial x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \left(1 - \frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{-\frac{\sqrt{3}}{2}\ \frac{\sqrt{3}}{2} - \left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right)} = \frac{0}{-1} = 0$$

$\sin 2x = 2\sin x \cos x$.

$2\sin x \cos x = \sin x$

$\sin x\,(2\cos x - 1) = 0$

Problem 5 (30 points) SET UP the integral(s) required to find the region between $y_1 = \sin(x)$ and $y_2 = \sin(2x)$ on the interval $[0, 2\pi]$



$$\overset{A}{\int_{0}^{\pi/3}} (\sin 2x - \sin x)\,dx + \overset{B}{\int_{\pi/3}^{\pi}} (\sin x - \sin 2x)\,dx + \overset{C}{\int_{\pi}^{5\pi/3}} (\sin 2x - \sin x)\,dx + \int_{5\pi/3}^{2\pi} (\sin x - \sin 2x)\,dx$$

Problem 6 (40 points) Now evaluate the answer you gave for problem 5 above.

Skip

Problem 7 (30 points) Evaluate the following:

$$\int \frac{x}{\sqrt{x-42}}$$

$u = x - 42$

$du = dx$

$u + 42 = x$

$$\int \frac{u+42}{\sqrt{u}}\,du$$

$$\int (u^{1/2} + 42u^{-1/2})\,du$$

$$\frac{u^{3/2}}{3/2} + 42\frac{u^{1/2}}{1/2} + C$$

$$\boxed{\frac{2}{3}(x-42)^{3/2} + 84(x-42)^{1/2} + C}$$