

Problem 1 (10 points) Name and Section Number

Problem 2 (20 points) SET UP the integral(s) to find the arc length of the following curve:

$$f(x) = x^3 e^x \quad x \in [0, 3]$$

$$f'(x) = x^3 e^x + 3x^2 e^x$$

Continuous everywhere

$$\int_0^3 \sqrt{1 + (x^3 e^x + 3x^2 e^x)^2} dx$$

Problem 3 (40 points) FIND the surface area when the following function is revolved about the y -axis.

$$f(t) = \frac{t^2}{4} \quad t \in [2, 4]$$

$$y = \frac{t^2}{4}$$

$$\frac{\partial t}{\partial y} = \frac{1}{\sqrt{y}} \text{ continuous on } [1, 4]$$

$$2\sqrt{y} = t$$

$$2\pi \int_1^4 2\sqrt{y} \sqrt{1 + \left(\frac{1}{\sqrt{y}}\right)^2} dy$$

$$2\pi \int_1^4 2\sqrt{y} \sqrt{1 + \frac{1}{y}} dy$$

$$4\pi \int_1^4 \sqrt{y+1} dy$$

$$\frac{4\pi (y+1)^{3/2}}{3/2} \Big|_1^4$$

$$\frac{8\pi}{3} (5)^{3/2} - \frac{8\pi}{3} (2)^{3/2}$$

Problem 4 (40 points) FIND the total surface area (so including the base!) when R (given below) is revolved about the y-axis.

$$f(x) = \sin(x), y = 0, x \in [0, \pi]$$

base is circle of radius π , so $\pi(\pi^2) = \pi^3$ units

$$y = \sin(x)$$

$$\sin^{-1}(y) = x$$

$$y \in [0, 1]$$

$$2\pi \int_0^1 \sin^{-1}(y) \sqrt{1 + \left(\frac{1}{1-y^2}\right)^2} dy$$

also not continuous at $y=1$

$$2\pi \int_0^1 \sin^{-1}(y) \sqrt{\frac{2-y^2}{1-y^2}} dy$$

Cannot actually evaluate based on what we have covered in class.