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MATH 2414 Quiz April 24, 2017

Problem 1 (10 points) Name and Section Number

Problme 2 (40 points) Write the Taylor series centered at zero for each of the following functions. Also, give the interval and radius of convergence.

$f_1(x) = \cos x$

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \qquad R = \infty \qquad I = (-\infty, \infty)$$

$f_2(x) = \sin x$

$$\sum_{k=0}^{\infty} \frac{(-1)^{k+1}(x)^{2k+1}}{(2k+1)!} \qquad I = (-\infty, \infty) \quad R = \infty$$

$f_3(x) = e^x$

$$\sum_{k=0}^{\infty} \frac{x^k}{k!} \qquad I = (-\infty, \infty) \qquad R = \infty$$

$f_4(x) = \dfrac{1}{1-x}$

$$\sum_{k=0}^{\infty} x^k \qquad I = (-1, 1) \qquad R = 1$$

Problem 3 (60 points) Find the Taylor series centered at zero for the following functions. Also, give the interval and radius of convergence.

$E(x) = \dfrac{1}{1-e^x}$

$$E(0) = \frac{1}{1-e^0} = \frac{1}{0}$$

not possible!

$B(x) = \dfrac{x}{1+x^2}$

$$B(x) = x\left(\frac{1}{1-(-x^2)}\right)$$

$$x\sum_{k=0}^{\infty}(-x^2)^k = \sum_{k=0}^{\infty}(-1)^k x^{2k+1} \qquad I = (-1,1) \qquad R = 1$$

$Y(x) = \dfrac{1}{1-x^e}$

$$Y(x) = \sum_{k=0}^{\infty}(x^e)^k = \sum_{k=0}^{\infty} x^{ek} \qquad I = (-1,1) \qquad R = 1$$

Problem 4 (40 points) Identify the function represented by the power series. Make sure to show work to demonstrate how you obtained your answer.

$\sum_{k=0}^{\infty} x^{5k}$ (A) $f(x) = \dfrac{1}{1-x^5}$ (B) $f(x) = \dfrac{5}{1-5x}$ (C) $f(x) = \dfrac{5}{1-x^5}$ (D) $f(x) = \dfrac{5}{1+x^5}$ (E) $f(x) = \dfrac{5}{1-5x}$

$$\sum_{k=0}^{\infty}(x^k)^5 = \sum_{k=0}^{\infty}(x^5)^k = \frac{1}{1-x^5} \qquad I = (-1,1)$$

Problem 5 (40 points) Identify the function represented by the power series. Make sure to show work to demonstrate how you obtained your answer.

$\dfrac{-1}{2}\sum_{k=0}^{\infty}\dfrac{1}{k}x^{10k}$ (A) $f(x) = \ln(\sqrt{1+10x})$ (B) $f(x) = -\ln(\sqrt{1-x^{10}})$ (C) $f(x) = \ln(1-x^{10})^2$ $\boxed{\text{(D) } f(x) = \ln(\sqrt{1-x^{10}})}$

The $\frac{1}{k}$ indicates an integral has occured.

$$-\frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{k}(x^{10})^k = -\frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{k}k(x^{10})^{k-1}10x^9 = -\frac{1}{2}\sum_{k=1}^{\infty}10x^9(x^{10})^{k-1}$$

$\boxed{\text{Derive}}$

$$= \frac{1}{2}\sum_{i=0}^{\infty}(-10x^9)(x^{10})^i = \frac{1}{2}(-10x^9)\sum_{i=0}^{\infty}(x^{10})^i.$$

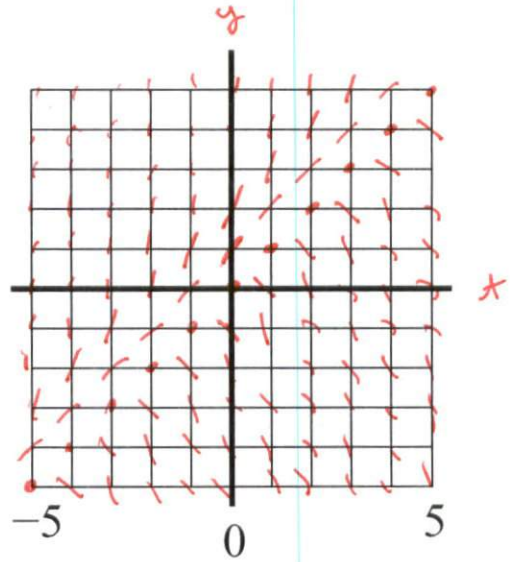
So $\frac{1}{1-x^{10}}$, mult. by $-10x^9$ and int., then multiply by $\frac{1}{2}$

$$\frac{1}{1-x^{10}} \to \frac{-10x^9}{1-x^{10}} \xrightarrow{\text{Int}} \ln(1-x^{10}) \to \frac{1}{2}\ln(1-x^{10}) \Rightarrow \text{answer}$$

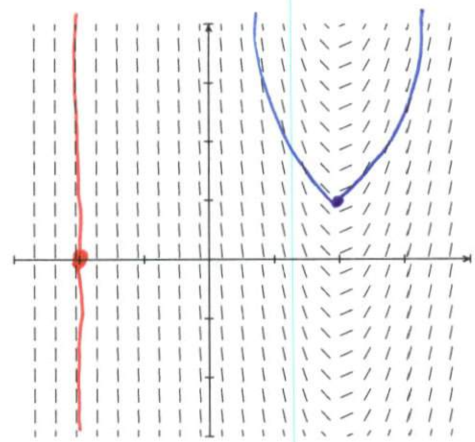
Problem 6 (30 points) Draw the slope field for the following differential equation.

$$\frac{dy(t)}{dt} = y - t$$

If $y = t$, $\frac{dy}{dt} = 0$



Problem 7 (30 points) Draw two different solutions in the given slope field, one starting at $(-2, 0)$ and the other starting at $(2, 1)$



Problem 8 (30 points) Find the specific solution of the IVP:

$$\frac{dy}{dx} = (x^2 + 1)e^{-y} \qquad y(1) = e$$

$dy\, e^y = (x^2+1)\, dx$

$e^y = \frac{x^3}{3} + x + C$

$e^e = \frac{1}{3} + 1 + C$

$e^e - \frac{4}{3} = C$

$\boxed{e^y = \frac{x^3}{3} + x + e^e - \frac{4}{3}}$

Problem 9 (30 points) Fill in the missing values in the table below given that $\frac{dy}{dt} = t^2 + 3t - 4$. Assume the rate of growth, given by $\frac{dy}{dt}$, is approximately constant over each time interval.

t	0	1	2	3	4
y	8	4	4	10	24
$\frac{dy}{dt}$	-4	0	6	14	

Problem 10 (40 points) What function is represented by the following, and where did we start? (So if the function is $2xe^{x^2}$, then we started with e^{x^2} and took a derivative)

$$\sum_{k=2}^{\infty} \frac{x^k}{k(k-1)} \xrightarrow{\text{Deriv}} \sum_{k=2}^{\infty} \frac{x^{k-1}}{k-1} \xrightarrow{\text{Deriv}} \sum_{k=2}^{\infty} x^{k-2} = \sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$$

So take $\frac{1}{1-x}$ and integrate twice. $\int \frac{1}{1-x}\,\partial x = -\ln(1-x) + C$

$$\int (-\ln(1-x))\,\partial x = -x\ln(1-x) + \int \frac{x}{1-x}\,\partial x = -x\ln(1-x) + \int \frac{1-u}{u}\,\partial u$$

parts, $u = \ln(1-x)$ $dv = dx$

$\partial u = \frac{-1}{1-x}$ $v = x$

$u = 1-x$
$du = -\partial x$
$x = 1-u$

$$\boxed{-x\ln(1-x) + \ln(1-x) + (1-x) + C}$$

Problem 11 (50 points) Find the Taylor series centered at zero for the following function. Also, give the interval and radius of convergence.

$$\ln\left(\frac{1-x}{1+x}\right)$$ by log rules, $\ln\left(\frac{1-x}{1+x}\right) = \ln(1-x) - \ln(1+x)$

Take derivatives $\longrightarrow$ $\frac{-1}{1-x} - \frac{1}{1+x} = (-1)\left(\frac{1}{1-x}\right) - \frac{1}{1-(-x)}$

So

$(-1)\sum_{k=0}^{\infty} x^k - \sum_{k=0}^{\infty} (-x)^k$ then Integrate each

$$\boxed{(-1)\sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1} - \sum_{k=0}^{\infty} \frac{(-1)^k (x)^{k+1}}{k+1}}$$