MATH 2414 ~~Quiz~~ April 13, 2017

130

Problem 1 (10 points) Name and Section Number

For each series below, determine if the series converges or diverges. If it converges, determine the sum if possible.

Problem 2 (20 points) The series is:

$$\sum_{k=1}^{\infty} \frac{-k}{(k+1)!}$$

$$\lim_{k\to\infty} \left| \frac{\frac{-(k+1)}{(k+1+1)!}}{\frac{-k}{(k+1)!}} \right|$$

$$\lim_{k\to\infty} \left| \frac{-(k+1)\;(k+1)!}{(k+2)!\;k} \right|$$

$$\lim_{k\to\infty} \left| \frac{-(k+1)}{(k+2)\,k} \right| = 0$$

so $0<1$, converges by ratio test.

$$\sum_{k=1}^{\infty} \frac{-k}{(k+1)!} = \sum_{k=1}^{\infty} \left(\frac{1}{(k+1)!} + \frac{-1}{k!} \right)$$

$$S_n = \frac{1}{(n+1)!} - \frac{1}{n!} + \frac{1}{n!} - \frac{1}{(n-1)!} + \dots - 1$$

$$S = \lim_{n\to\infty} \frac{1}{(n+1)!} - 1 = \boxed{-1}$$

Problem 3 (20 points) The series is:

$$\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

$$\lim_{k\to\infty} \left| \frac{\frac{((k+1)!)^2}{(2(k+1))!}}{\frac{(k!)^2}{(2k)!}} \right|$$

$$\lim_{k\to\infty} \left| \frac{(k+1)!\,(k+1)!}{(2k+2)!} \cdot \frac{(2k)!}{k!\;k!} \right|$$

$$\lim_{k\to\infty} \left| \frac{(k+1)(k+1)}{(2k+2)(2k+1)} \right| = \frac{1}{4} < 1$$ so converges by ratio test

Problem 4 (20 points) The series is:

$$\sum_{k=1}^{\infty} \frac{1}{(k^2-4)}$$

for $k \geq 3$

$$\frac{1}{k^2-4} > \frac{1}{k^2}$$

so no help

$$\sum_{k=1}^{\infty} \frac{-1}{k+2} + \frac{1}{k-2}$$

so Sum =

$$\lim_{k\to\infty} \left| \frac{k^2-4}{k^2} \right|$$

$$\lim_{k\to\infty} 1 - \frac{4}{k^2} = 1$$

since $\sum \frac{1}{k^2}$ is a p series, $p=2>1$ and hence converges, and we got a non-zero finite number, then

$\sum_{k=1}^{\infty} \frac{1}{k^2-4}$ converges by limit comparison

Problem 5 (60 points) Assume a given series is defined as $\sum_{k=1}^{\infty} r_k$. Fill-in-the-blanks with the appropriate word, phrase, inequality or mathematical statement. (That would include defining any ratios and /or terms used.)

- A geometric series is convergent if $|R| < 1$.
- A p-series is convergent if $p > 1$, for $\sum \frac{1}{x^p}$.
- The Integral Test can be used if the function associated with the given series is
  1. Positive
  2. decreasing
  3. continuous

  The Integral Test shows a series is convergent if The Integral $\int_1^{\infty} r_k \, dk$ converges.
- A series diverges by the Divergence Test if $\lim_{k\to\infty} r_k \neq 0$.
- The Root Test shows a series is convergent if $\lim_{k\to\infty} \sqrt[k]{r_k} < 1$.

  The Root Test shows a series is divergent if $\lim_{k\to\infty} \sqrt[k]{r_k} > 1$.

  The Root Comparison Test is inconclusive if $\lim_{k\to\infty} \sqrt[k]{r_k} = 1$.
- The Ratio Test shows a series is convergent if $\lim_{k\to\infty} \left|\frac{r_{k+1}}{r_k}\right| < 1$.

  The Ratio Test shows a series is divergent if $\lim_{k\to\infty} \left|\frac{r_{k+1}}{r_k}\right| > 1$.

  The Ratio Comparison Test is inconclusive if $\lim_{k\to\infty} \left|\frac{r_{k+1}}{r_k}\right| = 1$.
- The Direct Comparison Test shows a series is convergent if $\sum b_k$ converges AND $r_k \leq b_k$.

  The Direct Comparison Test shows a series is divergent if $\sum b_k$ diverges AND $b_k \leq r_k$.

  The Direct Comparison Test is inconclusive if $\sum b_k$ converges AND $r_k \geq b_k$ OR $\sum b_k$ diverges AND $r_k \leq b_k$.
- The Limit Comparison Test shows a series is convergent if $0 \leq L < \infty$; $L = \lim_{k\to\infty} \left|\frac{r_k}{b_k}\right|$ and b_k conv.

  The Limit Comparison Test shows a series is divergent if $0 < L \leq \infty$; $L = \lim_{k\to\infty} \left|\frac{r_k}{b_k}\right|$ and b_k diverges

  The Limit Comparison Test is inconclusive if none of the above 2 are met.