MATH 2414 Quiz March 30, 2017

**Problem 1 (10 points)** Name and Section Number

For each series below, determine if the series converges or diverges. If it converges, determine the sum if possible. HINT: for at least one, but not all, you should be able to determine the exact sum with maybe a little work.

**Problem 2 (40 points)** The series is:

$$\sum_{k=1}^{\infty} \frac{-k}{(k+1)!}$$

$$\lim_{k\to\infty} \left| \frac{\frac{-(k+1)}{(k+1+1)!}}{\frac{-k}{(k+1)!}} \right|$$

$$\lim_{k\to\infty} \left| \frac{-(k+1)}{(k+2)!} \cdot \frac{(k+1)!}{-k} \right|$$

$$\lim_{k\to\infty} \left| \frac{-(k+1)}{(k+2)(-k)} \right|$$

$= 0$; so $0 < 1$

converges by ratio test

As suggested:

$$\frac{-k}{(k+1)!} = \frac{E}{(k+1)!} + \frac{B}{k!}$$

So

$$\sum_{k=1}^{\infty} = \left( \frac{1}{(k+1)!} + \frac{-1}{k!} \right)$$

Telescopes

$$S_n = \frac{1}{(n+1)!} - \frac{1}{n!} + \frac{1}{n!} - \frac{1}{(n+1)!} \cdots - 1$$

$$S = \lim_{n\to\infty} \frac{1}{(n+1)!} - 1 = -1$$

**Problem 3 (40 points)** The series is:

$$\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

$$\lim_{k\to\infty} \left| \frac{\frac{(k+1)!\,(k+1)!}{(2k+2)!}}{\frac{k!\,k!}{(2k)!}} \right|$$

$$\lim_{k\to\infty} \left| \frac{(k+1)!\,(k+1)!}{(2k+2)!} \cdot \frac{(2k)!}{k!\,k!} \right|$$

$$\lim_{k\to\infty} \left| \frac{(k+1)(k+1)}{(2k+2)(2k+1)} \right| = \frac{1}{4}$$

$\frac{1}{4} < 1$ so converges by ratio test.

**Problem 4 (40 points)** The series is:

$$\sum_{k=1}^{\infty}\left(\frac{1}{\ln(k+1)}\right)^k$$

$$\lim_{k\to\infty} \sqrt[k]{\left(\frac{1}{\ln(k+1)}\right)^k}$$

$$\lim_{k\to\infty} \left|\frac{1}{\ln(k+1)}\right| = \frac{1}{\infty} = 0 < 1$$

$\boxed{\text{So this converges by root test}}$

**Problem 5 (40 points)** For this one, find the values of the parameter $R > 0$ for which the following series will converge:

$$\sum_{k=1}^{\infty}\frac{kR^k}{k+1}$$

$$\lim_{k\to\infty} \left|\frac{\frac{(k+1)R^{k+1}}{(k+1)+1}}{\frac{kR^k}{k+1}}\right| =$$

$$\lim_{k\to\infty} \left|\frac{(k+1)R^{k+1}}{(k+2)} \cdot \frac{(k+1)}{kR^k}\right|$$

$$\lim_{k\to\infty} \left|\frac{(k+1)(k+1)}{k(k+2)} R\right| = |R|$$

$\boxed{\text{so as long as } 0 < R < 1 \text{ converges by ratio test}}$