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MATH 2414 Quiz March 27, 2017

Problem 1 (10 points) Name and Section Number [illegible]

Problem 2 (30 points) Explain why the following statement is either true, false, or undetermined. If false or undetermined, provide examples of all cases.
If $\sum k^{-p}$ converges, then $\sum k^{-p+0.00001}$ converges.

$\sum k^{-p} = \sum \frac{1}{k^p}$; $\sum k^{-p+0.00001} = \sum \frac{1}{k^{p-.00001}}$

Undetermined. If $p > 1.00001$, then yes, but

if $1 < p < 1.00001$ then no, as p-series needs $p > 1$

Problem 3 (30 points) Determine if the following converges. If it does, find the limit if possible.

$$\sum_{k=1}^{\infty}\left[\arcsin\left(\frac{1}{k+1}\right) - \arcsin\left(\frac{1}{k}\right)\right]$$

$k=1$ $\arcsin(\frac{1}{2}) - \arcsin(1)$

$k=2$ $\arcsin(\frac{1}{3}) - \arcsin(\frac{1}{2}) + \arcsin(\frac{1}{2}) - \arcsin(1)$

So $S_n = \arcsin(\frac{1}{n+1}) - \arcsin(1)$

$\lim_{n\to\infty} =$

$\arcsin(0) - \arcsin(1)$

$= 0 - \pi/2 = \boxed{-\pi/2}$

Problem 4 (30 points) Determine if the following converges. If it does, find the limit if possible.

$$\frac{10}{3} + \frac{20}{9} + \frac{40}{27} + \frac{80}{81} + \cdots$$

$= 10(\frac{1}{3}) + 10(\frac{2}{9}) + 10(\frac{4}{27}) + 10(\frac{8}{81}) + \cdots$

$= \sum_{n=1}^{\infty} 10 \frac{2^{n-1}}{3^n}$

So this is a geometric

Series, $r = 2/3$ $|\frac{2}{3}| < 1$ so

it converges.

$S = \frac{\frac{10}{3}}{1 - \frac{2}{3}} = \frac{10/3}{1/3}$

$\boxed{S = 10}$

Problem 5 (30 points) Determine if the following converges. If it does, find the limit if possible.

$$\sum_{r=2}^{\infty} \frac{1}{r(\ln r)^P}$$

$u = \ln x$

$\partial u = \frac{1}{x} \partial x$

$f(x) = \frac{1}{x(\ln x)^P}$

cont; since $r=2$, after $r=3$, positive

$f'(x) =$ don't need; x gets bigger, $(\ln(x))^P$ gets bigger, so decreasing

$\lim_{E \to \infty} \int_3^E \frac{1}{x(\ln x)^P} \partial x = \left. \frac{(\ln x)^{-P+1}}{-P+1} \right|_3^E$ $\lim E \to \infty$

If $P<1$, diverges

$P=1$, $\ln|\ln(x)|$ diverges

$P>1$, converges.

Problem 6 (30 points) Determine if the following converges. If it does, find the limit if possible.

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{k+1}}$$

$f(x) = (x+1)^{-1/4}$

$f'(x) = -\frac{1}{4}(x+1)^{-5/4} = \frac{-1}{4(x+1)^{5/4}}$

always negative on $(0,\infty)$ so $f(x)$ decreasing

$f(x) > 0$ on $(0,\infty)$

$f(x)$ continuous on $(0,\infty)$

$\int_1^{\infty} (x+1)^{-1/4} \partial x = \lim_{E \to \infty} \int_1^E (x+1)^{-1/4} \partial x$

$\lim_{E \to \infty} \left. (x+1)^{3/4} \right|_1^E$

diverges, so the series diverges

Problem 7 (30 points) Determine if the following converges. If it does, find the limit if possible.

$$\sum_{k=1}^{\infty} \frac{k^e}{k^\pi} = \sum_{k=1}^{\infty} k^{e-\pi} = \sum_{k=1}^{\infty} \frac{1}{k^{\pi-e}}$$

$0 < \pi - e < 1$, so this p- series diverges.