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MATH 2414 Quiz March 23, 2017

Problem 1 (10 points) Name and Section Number

Problem 2 (80 points) Determine if the following converges. If it does, find the limit if possible.

$$\left\{1, \frac{e}{\pi}, \frac{e^2}{\pi^2}, \frac{e^3}{\pi^3}, \frac{e^4}{\pi^4}, \cdots\right\}$$

geo. $r = \frac{e}{\pi}$ $\left|\frac{e}{\pi}\right| < 1$, so converges

$e^{1/4}$

$$\left\{\sqrt{\left(1+\frac{1}{2n}\right)^n}\right\} = \left\{ \left(1+\frac{1}{2n}\right)^{n/2} \right\}$$

$\lim_{n\to\infty} \left(1+\frac{1}{2n}\right)^{n/2} = 1^\infty$

$\lim_{n\to\infty} y = \left(1+\frac{1}{2n}\right)^{n/2}$

$\lim_{n\to\infty} \ln(y) = \frac{n}{2} \ln\left(1+\frac{1}{2n}\right)$

$\lim_{n\to\infty} \ln(y) = \dfrac{\ln\left(1+\frac{1}{2n}\right)}{\frac{2}{n}} = \frac{0}{0}$

LH $\lim_{n\to\infty} \dfrac{\frac{1}{1+\frac{1}{2n}}\left(\frac{-1}{2n^2}\right)}{\frac{-2}{n^2}}$

$\lim_{n\to\infty} \dfrac{1}{4\left(1+\frac{1}{2n}\right)}$

$= \frac{1}{4}$

$$r_n = \frac{42^b + 6n!}{n! + 7^n}$$

$r_n = \dfrac{42^b}{n! + 7^n} + \dfrac{6n!}{n! + 7^n}$

as $n \to \infty$, growth rates suggest

$0 + 6 = 6$

$$\left\{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \cdots, \frac{n}{n+1}, \cdots\right\}$$

$\lim_{n\to\infty} \frac{n}{n+1} = 1$

Problem 3 (80 points) Let $\{B_n\}$ be defined by $B_{n+1} = (1/2)B_n^2$. Determine the limit of the sequence for each of the following four starting values.

(I) $B_1 = 0$

$B_2 = 0$, $B_3 = 0$, ... $B_n = 0$ for all n; hence limit $= 0$

$B_2 = \frac{1}{2}(2)^2 = 2$, $B_3 = \frac{1}{2}(2)^2 = 2$, ... so $\lim = 2$

(II) $B_1 = 2$

If $0 < B_1 < 2$, then $B_2 = \frac{1}{2}(B_1)^2 = B_1\left(\frac{B_1}{2}\right) < B_1$, as $B_1/2 < 1$

So we have a monotonic, decreasing sequence, bounded below by

(III) $0 < B_1 < 2$

zero, hence this also converges to zero

(IV) $B_1 > 2$

Diverges $B_2 = \frac{1}{2}B_1^2 = B_1\left(\frac{B_1}{2}\right)$, but $\frac{B_1}{2} > 1$, so we have an increasing sequence, unbounded, so diverges.

Problem 4 (30 points) Explain why the following is either true, false, or undetermined:
If $\{a_n\}$ converges, then $(-1)^n\{a_n\}$ MUST also converge.

Undetermined. If say $\{a_n\} = \frac{1}{n}$, then $(-1)^n\{a_n\}$ also converges to zero. But if $\{a_n\} = 1$, then $(-1)^n\{a_n\}$ fails to converge.

Problem 5 (60 points) SET UP (do not solve) the integral(s) required to determine if the following converges. (That would include the limit notation, etc) If it is not improper, then simply write OK.

$$\int_1^3 \frac{x}{\sin x}dx$$

OK. $\sin(0)$ or $\sin(\pi)$ a problem, but $0<1$ and $\pi>3$

$$\int_{42}^{\infty} \frac{dx}{x} = \int_{42}^{E} \frac{dx}{x}$$

$\lim E \to \infty$

$$\int_1^{\pi} \frac{\cos x}{x^2(x-3)}dx = \int_1^R \frac{\cos(x)}{x^2(x-3)}\,dx + \int_E^{\pi} \frac{\cos(x)}{x^2(x-3)}\,dx$$

$\lim R \to 3^-$ $\qquad$ $\lim E \to 3^+$

$$\int_{-3}^{4} \frac{dx}{x^2-x-12}$$

$x^2 - x - 12 = (x-4)(x+3)$

so

$$\int_{-3}^{4} \frac{dx}{x^2-x-12} = \int_R^0 \frac{dx}{x^2-x-12} + \int_0^E \frac{dx}{x^2-x-12}$$

$\lim R \to -3^+$ $\qquad$ $\lim E \to 4^-$

Could be any value in the interval $(-3,4)$