Math 2414 Final Review Answers

1.

a. $\frac{1}{4}$ units^2

b. $\frac{9}{2}$ units^2

c. $2\sqrt{2}$ units^2

d. $\frac{3}{4}-\frac{1}{2}(\ln 2)^2$ units^2

2.

a. $\frac{\pi}{2}$ units^3

b. $2\pi\left(e^2+1\right)$ units^3

c. 2π units^3

d. 16π units^3

e. 162π units^3

3.

a. $\ln\left(\sqrt{2}+1\right)$ units

b. $\frac{53}{6}$ units

4. a. $\frac{\pi}{6}\left(5\sqrt{5}-1\right)$ units^2

b. $\frac{\pi}{6}\left(17\sqrt{17}-1\right)$ units^2

5. $\frac{3}{\pi}$ J

6. 1125 J

7. 306.25 J

8. 68134.5π J

9.

a. $\frac{dy}{dx}=\coth(x)$

b. $f'(x)=x\cosh(x)$

10.

a. $-\frac{1}{2}\cos(2\theta)+\frac{1}{6}\cos^3(2\theta)+C$

b. $\frac{1}{3}\tan(3\theta)+\frac{1}{9}\tan^3(3\theta)+C$

c. $\frac{3}{8}x-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)+C$

d. $\frac{1}{9}\cos^9(x)-\frac{1}{7}\cos^7(x)+C$

e. $-\frac{1}{5}\cot^5(x)-\frac{1}{7}\cot^7(x)+C$

f. $\frac{1}{4}x^4\ln(x)-\frac{1}{16}x^4+C$

g. $-\frac{\ln(x)}{x}-\frac{1}{x}+C$

h. $-\frac{1}{5}x\cos(5x)+\frac{1}{25}\sin(5x)+C$

i. $-\frac{1}{3}x^2e^{-3x}-\frac{2}{9}xe^{-3x}-\frac{2}{27}e^{-3x}+C$

j. $x\tan^{-1}x-\frac{1}{2}\ln\left(1+x^2\right)+C$

11.

a. $-2\ln|x+1|+3\ln|x+2|+C$

b. $t+\ln|t|+C$

c. $x^2+4\ln|x^2-4|+C$

d. $-2\ln|x|+\frac{1}{x}+2\ln|x-1|+C$

e. $-\frac{1}{6}\ln|x-1|+\frac{1}{2}\ln|x-4|+\frac{1}{6}\ln|x+2|+C$

f. $2\ln|x|-\ln\left(x^2+2x+2\right)-2\tan^{-1}(x+1)+C$

g. $\ln\left(x^2+1\right)-2\ln|x+1|+2\tan^{-1}x+C$

12.

a. $\frac{1}{4}\left(\tan^{-1}(2x)+\frac{2x}{1+4x^2}\right)+C$

b. $x-2\tan^{-1}\left(\frac{x}{2}\right)+C$

c. $\ln\left|\frac{x}{2}+\frac{\sqrt{x^2-4}}{2}\right|-\frac{\sqrt{x^2-4}}{x}+C$

d. $\frac{1}{54}\left(\sin^{-1}(3x)-3x\sqrt{1-9x^2}\right)+C$

13.

a. $\frac{1}{2}$

b. 1

c. Diverges

d. $\frac{\pi}{2}$

e. $\frac{\pi}{5}$

14.

a. $\lim_{b\to\infty}\pi\int_4^b\frac{dx}{x}$

b. $\lim_{a\to 0^+}2\pi\int_a^4 x^{1/2}dx$

15.

a. 2π

b. $4\sqrt{2}\pi$

16.

a. $y=Ce^{\sin^{-1}x}$

b. $y=\left(\frac{1}{6}x^3+C\right)^2$

c. $y=\ln\left(\frac{e^{\sin x}}{1+Ce^{\sin x}}\right),\ C>-\frac{1}{e^{\sin x}}$

d. $y=\frac{2x^2}{1+x^2}$

17.

a. $a_n=2+5(n-1)=5n-3$

b. $a_n=\left(-\frac{2}{3}\right)^{n-1}$

18.

a. $\frac{1}{2}$

b. Diverges

c. Diverges

d. $\ln 2$

e. 0

f. $\frac{\pi}{2}$

g. $\frac{1}{3}$

h. Diverges

i. 0

j. $\frac{1}{e}$

19.

a. Geometric, converges to 5

b. Telescoping, converges to $\frac{3}{2}$

c. Geometric, diverges

d. Geometric, converges to $\frac{5}{6}$

e. *p*-series, converges

f. *p*-series, diverges

g. Integral Test, diverges

h. Direct Comparison Test, diverges

i. Limit Comparison Test, converges

j. Divergence Test, diverges

k. Divergence Test, diverges

l. Geometric, converges to $-\frac{45}{8}$

m. Direct Comparison Test, converges

n. Integral Test, converges

o. Ratio Test, diverges

p. Root Test, diverges

q. Telescoping, diverges

r. Limit Comparison Test, converges

20.

a. *p*-series, does not converge absolutely; Alternating Series Test, converges conditionally
b. Limit Comparison Test, diverges
c. Direct Comparison Test, converges absolutely
d. Geometric, converges absolutely
e. Divergence Test, diverges
f. Harmonic series, does not converge absolutely; Alternating Series Test, converges conditionally
g. Integral Test, converges absolutely
h. Ratio Test, converges absolutely
i. Limit Comparison Test, does not converge absolutely; Alternating Series Test, converges conditionally
j. Root Test, converges absolutely
k. Integral Test, does not converge absolutely; Alternating Series Test, converges conditionally
l. Direct Comparison Test, converges absolutely
m. Ratio Test or Integral Test, converges absolutely
n. Root Test, converges absolutely
o. Geometric, converges absolutely

21.

a. $I=[1,5),\ R=2,\ f'(x)=\sum_{n=1}^{\infty}\frac{(x-3)^{n-1}}{2^n},\ \int f(x)\,dx=\sum_{n=1}^{\infty}\frac{(x-3)^{n+1}}{n(n+1)2^n}+C$

b. $I=\{0\},\ R=0$ There is no reason to find the derivative and integral.

c. $I=(-5,13),\ R=9,\ f'(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n(x-4)^{n-1}}{3^{2n}},\ \int f(x)\,dx=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(x-4)^{n+1}}{(n+1)3^{2n}}+C$

d. $I=(-\infty,\infty),\ R=\infty,\ f'(x)=\sum_{n=1}^{\infty}\frac{x^{n-1}}{(n-1)!}=\sum_{n=0}^{\infty}\frac{x^n}{n!},\ \int f(x)\,dx=\sum_{n=0}^{\infty}\frac{x^{n+1}}{(n+1)!}+C=\sum_{n=-1}^{\infty}\frac{x^n}{n!}+C$

22.

a. $P_3=\frac{1}{2}-\frac{1}{4}(x-2)+\frac{1}{8}(x-2)^2-\frac{1}{16}(x-2)^3$

b. $P_3=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)-\frac{\sqrt{2}}{4}\left(x-\frac{\pi}{4}\right)^2-\frac{\sqrt{2}}{12}\left(x-\frac{\pi}{4}\right)^3$

23.

a. $\sum_{n=0}^{\infty}\frac{3^n x^n}{n!}$

b. $\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n}}{(2n)!}$

c. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}$

24.

a. semi-circle of radius 5 : $x^2 + y^2 = 25, \quad y \geq 0$

b. ellipse : $\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$

c. line segment between (2,1) and (1,3) on the line : $y = 5 - 2x$

d. half of the hyperbola : $x^2 - y^2 = 1$, the half corresponding to : $x \geq 1$

e. arc of parabola : $y = x^2$ corresponding to $-1 \leq x \leq 1$

25.

a. $3 + \ln 2$

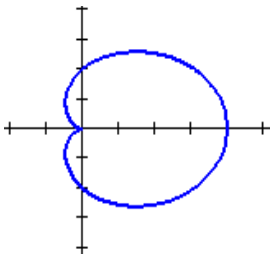
b. $\sqrt{10}$

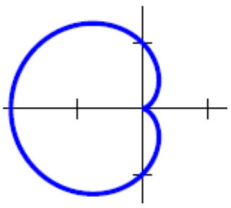
c. $\frac{8}{27}\left(37\sqrt{37} - 1\right)$

26.

a. 16

b. 8

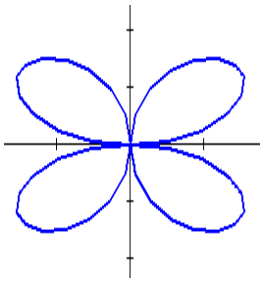


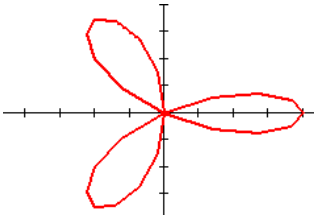


27.

a. $\frac{\pi}{2}$ units^2

b. $\frac{4\pi}{3}$ units^2

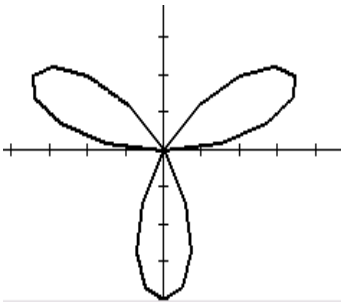


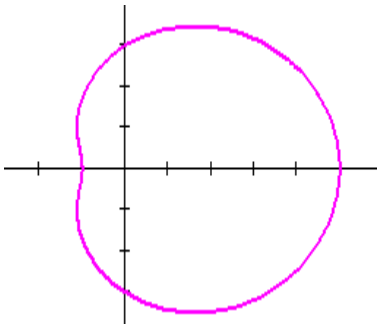


28.

a. 4π units^2

b. 11π units^2





29.

a. $\dfrac{dy}{dx} = \dfrac{-3}{8t}$

b. $\dfrac{dy}{dx} = -\dfrac{6}{5}\cot t$

30.

a. $m = \dfrac{-1+\sqrt{2}}{\sqrt{2}}$

b. $m = \dfrac{5\sqrt{3}}{11}$

31. $(x, y) = (50t, 64t - 16t^2),\ 0 \le t \le 4$