Review for MATH 2414 Final Exam

1. Sketch the region bounded between the given curves. Then compute the area of the region.
   a. $y = x^3,\ y = x,\ x = 0,\ x = 1$
   b. $x = y^2,\ x + y = 2$
   c. $y = \sin x,\ y = \cos x,\ x = 0,\ x = \pi$
   d. $y = \cosh x,\ y = x,\ x = 0,\ x = \ln 2$

2. Find the volume of the solid formed when:
   a. the region bounded by $y = \sqrt{x},\ x = 0,$ and $x = 1$ is rotated about the x-axis
   b. the region bounded by $y = e^x,\ x = 0,\ x = 2,$ and $y = 0$ is rotated about the y-axis
   c. the region bounded by $y = \sec x,\ y = 0,\ x = -\frac{\pi}{4},$ and $x = \frac{\pi}{4}$ is rotated about the x-axis
   d. the region bounded by $y = 4 - x,\ y = 4,$ and $y = x$ is rotated about the y-axis
   e. region bounded by $x = 2\sqrt{y},\ x = 0,$ and $y = 9$ is rotated about the y-axis

3. Find the length of each of the following curves.
   a. $y = \ln(\cos x)$ from $x = 0$ to $x = \frac{\pi}{4}$
   b. $y = \frac{x^3}{3} + \frac{1}{4x}$ from $x = 1$ to $x = 3$

4. Find the surface area generated when:
   a. $f(x) = \sqrt{x}$ for $0 \le x \le 1$ is revolved about $y = 0$
   b. $f(x) = x^2$ for $0 \le x \le 2$ is revolved about $x = 0$

5. When a particle is located at a distance x meters from the origin, a force of $\sin\left(\frac{\pi x}{3}\right)$ Newtons acts on it. How much work is done in moving the particle from $x = 1$ to $x = 2$?

6. If a force of 90 N stretches a spring 1 m beyond its natural length, how much *work* does it take to stretch the spring 5 m beyond its natural length?

7. A water trough has a length of 3 m and a semicircular cross section with a radius of 0.25 m. How much work is required to pump the water out of the trough when it is full?

8. An inverted conical tank with a height of 6 m and a base radius of 1.5 m is filled with milk, which has a density of $1030\ \text{kg/m}^3$. How much work is required to pump the milk out of the tank to the level of the top of the tank?

9. Find the derivative of the following functions.
   a. $y = \ln(\sinh x)$
   b. $f(x) = x\sinh x - \cosh x$

10. Evaluate the following integrals.
    a. $\int \sin^3(2\theta)\,d\theta$
    b. $\int \sec^4(3\theta)\,d\theta$
    c. $\int \sin^4 x\,dx$
    d. $\int \cos^6 x \sin^3 x\,dx$
    e. $\int \cot^4 x \csc^4 x\,dx$
    f. $\int x^3 \ln x\,dx$
    g. $\int \frac{\ln x}{x^2}\,dx$
    h. $\int x\sin(5x)\,dx$
    i. $\int x^2 e^{-3x}\,dx$
    j. $\int \tan^{-1} x\,dx$

11. Evaluate the following integrals.
   a. $\int \frac{x-1}{x^2+3x+2}\,dx$
   b. $\int \frac{t+1}{t}\,dt$
   c. $\int \frac{2x^3}{x^2-4}\,dx$
   d. $\int \frac{x+1}{x^3-x^2}\,dx$
   e. $\int \frac{x^2+2}{2(x+2)\left(x^2-5x+4\right)}\,dx$
   f. $\int \frac{4}{x\left(x^2+2x+2\right)}\,dx$
   g. $\int \frac{4x}{(x+1)\left(x^2+1\right)}\,dx$

12. Evaluate the following integrals.
   a. $\int \frac{dx}{\left(1+4x^2\right)^2}$
   b. $\int \frac{x^2}{x^2+4}\,dx$
   c. $\int \frac{\sqrt{x^2-4}}{x^2}\,dx$
   d. $\int \frac{x^2\,dx}{\sqrt{1-9x^2}}$

13. Either show that each of the following improper integrals converges and find its value, or show that it diverges.
   a. $\int_2^{\infty} \frac{dx}{x^2}$
   b. $\int_0^{\infty} xe^{-x}\,dx$
   c. $\int_{-1}^{1} \frac{dx}{x}$
   d. $\int_0^{1} \frac{dx}{\sqrt{1-x^2}}$
   e. $\int_{-\infty}^{\infty} \frac{dx}{1+25x^2}$

14. Use the definition of the improper integral to setup an expression that correctly defines the volume generated when each region is revolved around the axis indicated.
   a. region in QI below $y=x^{-1/2}$ and to the right of $x=4$ is revolved about the x-axis
   b. region in QI below $y=x^{-1/2}$ and to the left of $x=4$ is revolved about the y-axis

15. Find the volume, if it exists, of the described solid of revolution
   a. the region bounded by $y=x^{-3}$ and $y=0$ for $x \ge 1$, revolved about the y-axis
   b. the region bounded by $y=(x+1)^{-3/2}$ and $y=0$ for $-1 < x \le 1$, revolved about $x=-1$

16. Solve the following differential equations.
   a. $\frac{dy}{dx}=\frac{y}{\sqrt{1-x^2}}$
   b. $\frac{dy}{dx}=x^2\sqrt{y}$
   c. $\sec x \cdot \frac{dy}{dx}=e^{y-\sin x}$
   d. $\frac{dy}{dx}=\frac{y^2}{x^3},\ y(1)=1$

17. Give the formula for the n^{th}-term, a_n, of each sequence.
   a. $\{2, 7, 12, 17, \ldots\}$
   b. $\left\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots\right\}$

18. Determine if the following sequences converge or diverge. When possible, find the limit.

a. $a_n = \dfrac{n}{2n+1}$

b. $a_n = \dfrac{n^3}{n+1}$

c. $a_n = \dfrac{(-1)^n n^3}{n^3 + 2n^2 + 1}$

d. $a_n = \ln(2n+1) - \ln(n)$

e. $a_n = \dfrac{\cos n}{n^2}$

f. $a_n = \tan^{-1} n$

g. $a_n = \sqrt{\dfrac{n+1}{9n+1}}$

h. $a_n = \cos\left(\dfrac{n\pi}{2}\right)$

i. $a_n = ne^{-n}$

j. $a_n = \left(1 - \dfrac{1}{n}\right)^n$

19. Determine if the following series converge or diverge. If a series is a convergent geometric or telescoping series, give its sum. In each case, state the test(s) used.

a. $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n}}{5^n}$

b. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$

c. $\displaystyle\sum_{k=0}^{\infty} \frac{(-5)^k}{3^{k+1}}$

d. $\displaystyle\sum_{k=2}^{\infty} \left(\frac{2}{3^k} + \frac{1}{2^k}\right)$

e. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^3}$

f. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{\sqrt[3]{n}}$

g. $\displaystyle\sum_{k=2}^{\infty} \frac{2}{k \ln k}$

h. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2}{n^3 - 4}$

i. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2 - 2n + 1}{4n^4 + 3n^3 - n - 5}$

j. $\displaystyle\sum_{k=0}^{\infty} \frac{4k}{k+2}$

k. $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{k^3}{k^2 + 1}$

l. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 3^{n+1}}{5^{n-1}}$

m. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2 + 1}$

n. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{\sqrt{n}}$

o. $\displaystyle\sum_{k=1}^{\infty} \frac{k^k}{k!}$

p. $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{k^{10}}$

q. $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$

r. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$

20. Determine if the following series converge absolutely, converge conditionally, or diverge. State the test(s) used.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

b. $\displaystyle\sum_{n=0}^{\infty} \frac{2n}{4n^2 + 1}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2}$

d. $\displaystyle\sum_{n=0}^{\infty} \frac{4 + 3^n}{5^n}$

e. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{2n+1}$

f. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4 + 1}$

g. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3}$

h. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

i. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{n}}{n+2}$

j. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$

k. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$

l. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2 + 3^n}$

m. $\displaystyle\sum_{n=1}^{\infty} n\, e^{-n^2}$

n. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n}{1 + 8n}\right)^n$

o. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$

21. Determine the radius and interval of convergence for each of the following power series. Then find the derivative and integral of each series.

a. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n2^n}$ b. $\sum_{n=0}^{\infty} n!\,x^n$ c. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-4)^n}{3^{2n}}$ d. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

22. Find the first four nonzero terms of the Taylor series centered at the given a for the functions.

a. $f(x) = \frac{1}{x},\ a = 2$ b. $f(x) = \sin x,\ a = \frac{\pi}{4}$

23. Find the Maclaurin series of the following functions.

a. $f(x) = e^{3x}$ b. $f(x) = \cos\left(x^2\right)$ c. $f(x) = \sin(2x)$

24. Identify the type (arc of parabola, circle, ellipse, hyperbola, line segment, etc.) of each of the following parametric curves.

a. $(x, y) = (5\cos t, 5\sin t),\ 0 \le t \le \pi$

b. $(x, y) = (2\cos t, 3\sin t),\ 0 \le t \le 2\pi$

c. $(x, y) = (2-t, 1+2t),\ 0 \le t \le 1$

d. $(x, y) = (\sec t, \tan t),\ -\frac{\pi}{2} < t < \frac{\pi}{2}$

e. $(x, y) = \left(t, t^2\right),\ -1 \le t \le 1$

25. Find the length of the following parametric curves.

a. $(x, y) = \left(2\sqrt{2}\,t, t^2 - \ln t\right),\ 1 \le t \le 2$

b. $(x, y) = (2+t, 1+3t),\ 0 \le t \le 1$

c. $(x, y) = \left(t^2, t^3\right),\ 0 \le t \le 4$

26. Sketch each of the following cardioids. Then compute the length of each curve.

a. $r = 2 + 2\cos\theta$ b. $r = 1 - \cos\theta$

27. Sketch each of the following rose curves. Then compute the area enclosed by <u>one</u> of the loops.

a. $r = 2\sin 2\theta$ b. $r = 4\cos(3\theta)$

28. Sketch each of the following polar curves. Then compute the area enclosed by each curve.

a. $r = 4\sin(3\theta)$ b. $r = 3 + 2\cos\theta$

29. Find $\frac{dy}{dx}$ given

a. $(x, y) = \left(4t^2, -3t\right)$ for $-2 \le t \le 3$ b. $(x, y) = (5\cos t, 6\sin t)$ for $0 \le t \le 2\pi$

30. Determine the slope of the curve at the given point

a. $r = 2 - \sin\theta$, at $\theta = \frac{\pi}{4}$ b. $r = 3\cos 4\theta$ at $\theta = \frac{\pi}{6}$.

31. A projectile is launched from a point (origin) with an initial velocity that has horizontal component of 50 ft/s and vertical component of 64 ft/s. Assume only the gravitational force affects the motion of the object, g = 32 ft/s^2. Give a parametrization of the trajectory of the object.

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