Math 2413 Memo Question set two limits

1. Sometimes students try to think about limits as *As you get closer and closer to the limit point the function gets closer and closer to the limit value.* Explain why that does or does not work for each of the following:

   A As x increases to 2, $f(x) = x^2$ gets closer and closer to 4, so the limit at $x = 2$ of $f_A(x) = 4$.

   B As x increases to 100, $f(x) = 1/x$ gets closer and closer to 0, so the limit as x goes to 100 of $f_B(x) = 0$

2. Sometimes students try to think about limits as *As you get closer and closer to the limit point the function gets closer and closer to the limit value.* Explain why that does or does not work for each of the following:

   A As x increases to 2, $f(x) = \sqrt{x}$ gets closer and closer to 1.414, so the limit at $x = 2$ of $f_A(x) = 1.414$.

   B As x increases to 1000, $f(x) = \sqrt[3]{x}$ gets closer and closer to 10, so the limit as x goes to 1000 of $f_B(x) = 10$

3. Suppose $f(r) = R, f(b) = B, r, b > 0$. Explain what the ARC between $-r$ and $-b$ would be for $f(x)$ if f is an even function. (consider all bases of $B > b, B < b$ etc.)

4. Suppose $f(r) = R, f(b) = B, r, b > 0$. Explain what the ARC between $-r$ and $-b$ would be for $f(x)$ if f is an odd function. (consider all bases of $B > b, B < b$ etc.)

5. Is there a number r such that the following limit exists? Explain how you found r. Also, consider the $\lim_{x\to 0}$ NOW what value(s) of r work? What is the difference?

$$\lim_{x\to 3} \frac{2x^2 - 3rx + x - r - 1}{x^2 - 2x - 3}$$

6. Let $f(x)$ be defined as below. Determine the two given limits, and explain your answer.

$$\lim_{x\to 0} f(\sin x) \quad \lim_{x\to 0} \cos(f(x)) \quad \lim_{x\to 0} f(f(x)) \quad f(x) = \begin{cases} 0 & x \neq 0 \\ \pi & x = 0 \end{cases}$$

7. Let $f(x)$ be defined as below. Determine the two given limits, and explain your answer.

$$\lim_{x\to 0} f(f(x)) \quad \lim_{x\to 0} \sin(f(x)) \quad \lim_{x\to 0} f(\cos(x)) \quad f(x) = \begin{cases} \pi & x \neq 0 \\ 0 & x = 0 \end{cases}$$

8. Consider the angle θ in standard position in a unit circle, $-\pi/2 < \theta < \pi/2$. Show that $|\sin\theta| < |\theta|$ HINT: Draw the triangle for θ in the unit circle.

9. Consider the angle θ in standard position in a unit circle, $-\pi/2 < \theta < \pi/2$. Show that $0 \leq 1 - \cos\theta < |\theta|$ HINT: Draw the triangle for θ in the unit circle.

10. Find functions f and g such that the following are true. Explain your thinking how you found the function.

$$\lim_{x\to 1} f(x) = 0 \quad \lim_{x\to 1}(f(x)g(x)) = 5$$