Math 2413 – Final Exam Review Answers

1.
   a. $-\frac{\pi}{3}$
   b. $\frac{5\pi}{6}$
   c. $\frac{\sqrt{4-y^2}}{2}$

2.
   a. $\frac{3}{5}$
   b. 32

3.
   a. $x=1,\ x=-2$
   b. $x=-\frac{3}{2}$

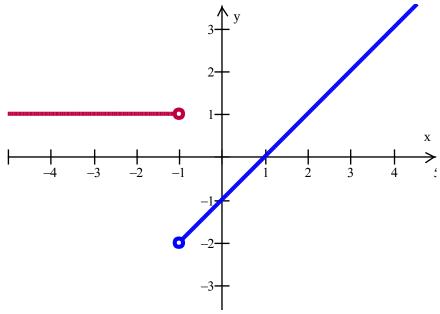
4.
   a. $\lim_{x\to -1^-} f(x)=3$
   b. $\lim_{x\to -1^+} f(x)=3$
   c. $\lim_{x\to -1} f(x)=3$
   d. $\lim_{x\to 2^-} f(x)=0$
   e. $\lim_{x\to 2^+} f(x)=-1$
   f. $\lim_{x\to 2} f(x)$ does not exist
   g. f is discontinuous at $x=1$ because $\lim_{x\to 1} f(x)\neq f(1)$; f is discontinuous at $x=2$, because $\lim_{x\to 2} f(x)$ does not exist.
   h. f is not differentiable at $x=-1$ because there is a cusp; f is not differentiable at $x=1$ because the function is discontinuous; f is not differentiable at $x=2$ because the function is discontinuous.

5.
   a. $\frac{4}{\pi}$
   b. $-\frac{3\sqrt{3}}{\pi}$

6.
   a. $\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=3x^2$
   b. $\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}=e^x$
   c. $\lim_{h\to 0}\frac{\tan^{-1}(x+h)-\tan^{-1}(x)}{h}=\frac{1}{x^2+1}$
   d. $\lim_{x\to 2} x^3-5x^2+4=-8$
   e. $\lim_{x\to -1}\frac{x^2-2x-3}{x+1}=-4$
   f. $\lim_{x\to 2^+}\frac{3+x}{2-x}=-\infty$
   g. $\lim_{x\to 3^-}\frac{|x-3|}{x-3}=-1$
   h. $\lim_{x\to\infty}\frac{9x^5+50x^2+800}{x^5-1000}=9$
   i. $\lim_{h\to 0}\frac{(4+h)^2-16}{h}=8$
   j. $\lim_{h\to 0}\frac{3-\sqrt{9+h}}{h}=-\frac{1}{6}$
   k. $\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}=-1$
   l. $\lim_{x\to \frac{\pi}{2}}\frac{\ln(\csc x)}{\left(x-\left(\frac{\pi}{2}\right)\right)^2}=\frac{1}{2}$
   m. $\lim_{x\to 0^+}\frac{\ln\left(e^x-1\right)}{\ln x}=1$
   n. $\lim_{x\to 0^+}\left(1+\frac{1}{x}\right)^x=1$
   o. $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x=e^{-2}$
   p. $\lim_{x\to\infty}\left(x^{-5}\ln x\right)=0$
   q. $\lim_{\theta\to -\infty}\frac{\cos\theta}{3\theta}=0$
   r. $\lim_{x\to 1}\frac{x^3-1}{x^2-1}=\frac{3}{2}$

7.
   a. $a=1,\quad b=-\frac{18}{5}$
   b. $b=0,\ b=\frac{1}{9}$

8.



9.

a. Continuous everywhere; Not differentiable at $x=2$
b. Continuous everywhere; Not differentiable at $x=0$
c. Continuous everywhere; Differentiable everywhere
d. Continuous everywhere; Not differentiable at $x=1$
e. Continuous everywhere; Differentiable everywhere

10.

a. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=4x-1$

b. $y=3x+1$

11.

a. $f'(x)=2x+2x^{-3}+\dfrac{2}{3}x^{-1/3}$

b. $f'(x)=\dfrac{22}{(x+9)^2}$

c. $f'(x)=6x(3x+5)(2x^3+x^2)^2(11x^2+19x+5)$

d. $f'(x)=4\sec(4x)\tan(4x)$

e. $f'(x)=x\left(2\ln(3x-1)+\dfrac{3x}{3x-1}\right)$

f. $f'(x)=15e^{5x-4}$

g. $f'(x)=\dfrac{2x}{\sqrt{1-x^4}}$

h. $f'(x)=-\csc x\cot x$

i. $f'(x)=\dfrac{\cos x+x\sin x}{\cos^2 x}-\dfrac{\sin x-x\cos x}{\sin^2 x}$

j. $f'(x)=6\tan(3x)\sec^2(3x)\sec(x)+\sec(x)\text{ta}$

k. $f'(x)=\dfrac{4x}{(\ln 3)(2x^2+1)}+6^{3x^3}(\ln 6)(9x^2)$

12.

a. $f^{(4)}(x)=120x$

b. $f^{(4)}(x)=\dfrac{72}{(x-2)^5}$

c. $f^{(4)}(x)=-\sin x$

d. $f^{(4)}(x)=625e^{5x}$

e. $f^{(4)}(x)=256\cos(4x)$

13.

a. $h'(1)=\dfrac{37}{2}$

b. $h'(0)=-\dfrac{9}{2}$

c. $h'(0)=12$

d. $h'(1)=-294$

14.

a. -1

b. $-\dfrac{5}{2}$

15. $m_{\tan}=2$

16. $v(t)=4t^3-12t^2+8$, $a(t)=12t^2-24t$

17. $x=-5$, $x=1$

18. $y=\sqrt{\pi}x$

19.

a. $\dfrac{dy}{dx}=\dfrac{-2x-3y}{3x+2y}$

b. $\dfrac{dy}{dx}=\dfrac{2xye^{2x}-y}{x}$

c. $\frac{dy}{dx} = \frac{\sec y}{2y - x\sec y \tan y}$

d. $\frac{dy}{dx} = (\ln x + 1)x^x$

e. $\frac{dy}{dx} = \frac{2}{\sqrt{1-(2x+1)^2}}$

f. $\frac{dy}{dx} = 2x\tan^{-1}(3x) + \frac{3x^2}{1+9x^2}$

20. $\frac{3}{80} \frac{\text{rad}}{\text{sec}}$

21. $-\frac{9}{4} \frac{\text{ft}}{\text{min}}$

22. $54\pi \ \frac{\text{in}^3}{\text{min}}$

23.

a. The critical value is $x = \frac{\pi}{4}$.
There is an absolute minimum of 0 at $x = 0, \frac{\pi}{2}$.
There is an absolute maximum of $\frac{1}{2}$ at $x = \frac{\pi}{4}$.

b. The critical values are $x = 0, \frac{4}{5}$.
There is an absolute minimum of -2 at $x = -1$.
There is an absolute maximum of 0 at $x = 0, 1$.

c. The critical value is $x = 1$.
There is an absolute minimum of 0 at $x = 1$.
There is an absolute maximum of 3 at $x = -2$.

24.

a. Increasing: $(0,8)$
Decreasing: $(-\infty,0)\cup(8,\infty)$

b. Increasing: $(-\infty,-4)\cup(2,\infty)$
Decreasing: $(-4,2)$

c. Increasing: Nowhere
Decreasing: $(-\infty,-2)\cup(-2,\infty)$

25.

a. The points of inflection are $\left(-2,-\frac{96}{5}\right)$ and $\left(2,-\frac{96}{5}\right)$.
Concave up: $(-\infty,-2)\cup(2,\infty)$
Concave down: $(-2,0)\cup(0,2)$

b. There are no points of inflection.
Concave up: $(-\infty,\infty)$
Concave down: Nowhere

26.

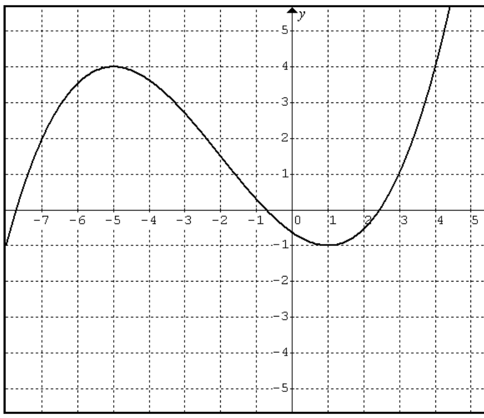
a. $a = 3$ and $b = -9$

b. Since $f''(1) = 12$ a relative minimum occurs at $x = 1$.
Since $f''(-3) = -12$ a relative maximum occurs at $x = -3$.

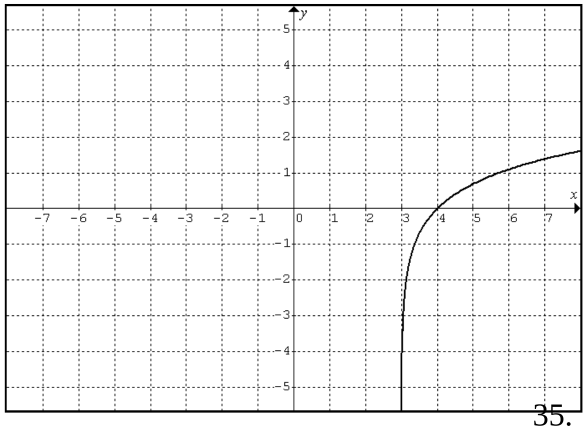
c. There is a relative minimum of 2 at $x = 1$.
There is a relative maximum of 34 at $x = -3$.

27. $x = -3$ and $x = 2$

28.



29.



30. The cable should begin to run underwater 1 mile from point *A*.

31. $\frac{5}{2}"\times 10"\times 10"$

32. The man should row to a point $\frac{1}{4}$ mile from the town.

33. The stake should be placed 6 meters from the 3-meter pole.

34.

a. $\int \csc\theta\cot\theta\, d\theta = -\csc\theta + C$

b. $\int\left(-18t^2+5t-\frac{4}{t}\right)dt = -6t^3+\frac{5t^2}{2}-4\ln|t|+C$

c. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x)+C$

d. $\int \frac{x\,dx}{\sqrt{1-x^2}} = -\sqrt{1-x^2}+C$

e. $\int (x^2-2)e^{x^3-6x}\,dx = \frac{1}{3}e^{x^3-6x}+C$

f. $\int \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\ln|x^3+1|+C$

g. $\int \sec^2(2t)\tan(2t)\,dt = \frac{\tan^2(2t)}{4}+C$

h. $\int \frac{\sin^{-1}(7x)}{\sqrt{1-49x^2}}\,dx = \frac{\left(\sin^{-1}(7x)\right)^2}{14}+C$

i. $\int \frac{1}{9x^2+4}\,dx = \frac{1}{6}\tan^{-1}\left(\frac{3}{2}x\right)+C$

j. $\int_{-\pi/3}^{\pi/3} \sec^2\theta\, d\theta = 2\sqrt{3}$

k. $\int_4^{16} \frac{1}{\sqrt{x}}\,dx = 4$

l. $\int_0^1 \frac{4}{1+\theta^2}\,d\theta = \pi$

m. $\int_0^1 \left(5x^3+ax\right)dx = \frac{2a+5}{4}$

35.

a. $\sum_{k=1}^{9} -\frac{k}{5} = -9$

b. $\sum_{k=1}^{14}\left(3-k^2\right) = -973$

c. $\sum_{k=1}^{5} k\left(6k^2+5\right) = 1425$

36. $\Delta x = \frac{4}{n}$

partition: $\left[0,\frac{4}{n}\right],\left[\frac{4}{n},\frac{8}{n}\right],\left[\frac{8}{n},\frac{12}{n}\right],\ldots,\left[\frac{4(n-1)}{n},4\right]$

$\overline{x}_k = \frac{4k}{n}$ and $f\left(\overline{x}_k\right) = \frac{16k}{n}+1$

$\int_0^4 (4x+1)\,dx = 36$

37.

a. $\frac{6x\left(9x^4-1\right)}{\sqrt{3x^2+1}}$

b. $-x^3\cos^2 x$

c. $\sec^3 x\tan^2 x$

38.

a. $\frac{157}{6}$ units^2

b. $\frac{3-\sqrt{3}}{2}$ units^2

c. $\frac{64}{15}$ units^2

39. $s(t) = t^2-\sin t+5t+5$