Section #5.3 Definite Integral as Area
MATH 2313

If the graph of $f(x)$ is positive, the area under the curve is $\int_a^b f(x)\,dx$

Discussion: Find $\int_{-2}^2 (x^2 - 4)\,dx$

What does the negative represent?

Examples:

1. Given the graph of $f(x)$ answer the following:

   (a) What is the TOTAL area?
   
   (b) Find $\int_0^4 f(x)\,dx$

Find $\int_{-2}^2 -\sqrt{4 - x^2}\,dx$

What does the negative represent?

Given the graph of $f(x)$ answer the following:

Find $\int_{-4}^5 f(x)\,dx$

What is the TOTAL area?
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Area between two curves
To help with these problems, it is advised to always graph the functions.

2. Find the area bounded by \( y = \sqrt{x} \) and \( y = x \) between \( x = 0 \) and \( x = 1 \).

How do you find \( \int_{a}^{b} f(x) \, dx \)?

- using RIEMANN
- using \( \text{fnInt(expression, X, a,b) } \) command under \( \text{MATH} \) option 9

3. Find the area of the region bounded by \( y = x^2 \) and \( y = x^5 \)

4. Find the area between \( f(x) = x^2 - 4 \) and \( g(x) = 5 \)
5. Find the area bounded $f(x) = x^3 - 9x$ and the x-axis

6. Find the area between $f(x) = x + 2, g(x) = x^2 - 4$, $x = -1, x = 2$

7. Find the area between $f(x) = x + 2, g(x) = x^2 - 4$, $x = -1, x = 4$
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Summary

- If want area between curve and x-axis \( \int_{a}^{b} (f(x) - 0) \, dx \)
- If want area between two curves \( \int_{a}^{b} (top - bottom) \, dx \)
- If want area between two curves and they intersect:
  \( \int_{a}^{c} (g(x) - f(x)) \, dx + \int_{c}^{b} (f(x) - g(x)) \, dx \)

8. Given the following graph of \( f(x) \), find:

(a) \( \int_{0}^{2} f(x) \, dx = \)

(b) Area between \( f(x) \) and x-axis between \([0, 2]\)

(c) \( \int_{0}^{5} f(x) \, dx = \)

(d) Area between \( f(x) \) and x-axis between \([0, 5]\)