Section #4.7 - Logistic Growth
MATH 2313

Def: Logistic function has the form: \( P = f(x) = \frac{L}{1 + Ce^{-kt}} \)

\[
\begin{align*}
  f(x) &= \frac{10}{1 + 9e^{-t}} \\
  f(x) &= \frac{10}{1 + 9e^{-2t}} \\
  f(x) &= \frac{10}{1 + 9e^{-0.5t}}
\end{align*}
\]

Note: that as \( k \) increases, the curve approaches the asymptote more rapidly. So \( k \) affects the steepness.

The value of \( C \), on the other hand:

\[
\begin{align*}
  f(x) &= \frac{10}{1 + 5e^{-t}} \\
  f(x) &= \frac{10}{1 + 10e^{-t}} \\
  f(x) &= \frac{10}{1 + 2.5e^{-t}}
\end{align*}
\]

Properties of a logistic function

- The value \( L \) is called __________ and represents the largest population an environment can support.
- The inflection point, where the concavity changes, the slope is changing the most rapidly. This point is called the _____ of diminishing returns. It occurs where \( P = \frac{L}{2} \)
The logistic function is approximately exponential for small values of $x$, with growth rate $k$.

$$f(t) = \frac{10}{1 + e^{-t}}$$

Some people believe that the earth's population cannot exceed 40 billion people. Let $P = \text{pop. in billion}$ and $t = \text{years since 1990}$.

$$P(t) = \frac{45}{1 + 11e^{-0.08t}}$$

1. What does this model predict for the maximum population?

2. When would the population reach 32.9 million?

3. When is the population changing the most rapidly?

In Wilson Corners, population 2000, a rumor spreads according to the logistic model. If 5 people know the rumor at 4 PM and 160 people have heard it by 5 PM, how many will have heard the rumor by 6 PM?