Section #2.3– Interpretations of the Derivative

MA TH 2313

Discussion: Suppose we’re given the position function \( s(t) \) which measures the distance of an object against time. What does the derivative of \( s(t) \) represent?

Suppose you’re given the velocity function \( v(t) \), what does the derivative of \( v(t) \) represent?

Examples:

1. If \( g(v) \) is the fuel efficiency of a car going at \( v \) miles per hour (i.e., \( g(v) = \) the number of miles per gallon at \( v \) mph), what are the units of \( g'(55) \)? What is the practical meaning of the statement \( g'(55) = -0.54 \)?

   (a) Interpret the statement:
   
   \( g(25) = 11 \)
   
   \( g'(25) = 2 \)

2. The number of new subscriptions to a newspaper, \( y \), in a month is a function of the amount, \( x \), in dollars spent on advertising in that month, so \( y = f(x) \).

   (a) Interpret the statement:
   
   \( f(250) = 180 \)
   
   \( f'(250) = 2 \)

   (b) Use the statements given in part a to estimate:
   
   \( f(251) \)
   
   \( f(260) \)

3. Let \( P \) be the total petroleum reservoir on earth in the year \( t \). (In other words, \( P \) represents the total quantity of petroleum, including what’s not yet discovered, on earth at time \( t \).) Assume that no new petroleum is being made and the \( P \) is measured in barrels. What are the units of \( \frac{dP}{dt} \)? What is the meaning of \( \frac{dP}{dt} \)? What is its sign? How would you set about estimating this derivative in practice? What would you need to know to make such an estimate?

Different Notations

1. Leibniz’s used \( \frac{dx}{dt} \) introduced in 1670s. remember this is "a small change in \( x \) over a small change in \( t \)."

   Don’t cancel the "d". That would be like cancelling the 2’s in \( \frac{27}{28} \)

2. Lagrange used \( x'(t) \) or could also write \( \frac{d}{dt}(x) \) meaning derivative of \( x \) with respect to \( t \).