Section #2.1–Instantaneous Rate of Change
MATH 2313

Remember rate of change = \( \frac{f(b)-f(a)}{b-a} \) (slope between \( a \) and \( b \))

Def: instantaneous velocity of an object at time \( t \) is defined to be the limit of the average velocity of the object over shorter and shorter time intervals.

(Could be thought of as the slope of the tangent line)

Discussion: What does the speedometer (possible) read on a car going 30 miles in 1 hour?
One mile in 2 minutes?
44 feet in one second?

Example:

1. An object is projected upward from the ground at \( t = 0 \) and its distance above the ground at time \( t \) is given in the following table:

\[
\begin{array}{c|c|c|c|c|c}
\hline
\text{\( t \) (sec)} & 0 & 1 & 2 & 3 & 4 & 5 \\
\text{\( g \) (ft)} & 0 & 72 & 112 & 120 & 90 & 40 \\
\hline
\end{array}
\]

(a) Find the average velocity between
[1,2]
[2,3]
[3,4]
[1,3]
[0,5]

(b) Estimate the velocity at \( t = 2 \) and at \( t = 3 \).
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Def: The instantaneous rate of change of \( f \) at \( a \) (also called the rate of change of \( f \) at \( a \)) is defined to be the limit of the average rates of change of \( f \) over shorter and shorter intervals around \( a \).

Def: The derivative of \( f \) at \( a \) written \( f'(a) \) is

Example:

2. What’s the velocity at \( t = 2 \) seconds?

<table>
<thead>
<tr>
<th>( t ) (sec)</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s(t) ) (ft)</td>
<td>72</td>
<td>95.539</td>
<td>112</td>
<td>119.13</td>
<td>120</td>
</tr>
</tbody>
</table>

3. \( t \) (sec) | 1.9 | 1.99 | 1.999 | 2 | 2.001 | 2.01 | 2.1 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) (ft)</td>
<td>109.44</td>
<td>111.7584</td>
<td>111.976</td>
<td>112</td>
<td>112.0240</td>
<td>112.2384</td>
<td>114.24</td>
</tr>
</tbody>
</table>

4. If the quantity of a drug in the bloodstream at \( t \) seconds is \( Q(t) = 25e^{-0.2t} \), estimate the rate of change of the quantity in the bloodstream at \( t = 4 \) seconds.

5. Estimate \( f'(2) \) for \( f(x) = 2 - x^3 \). Use the table method, with \( \Delta x = 0.1 \) and \( \Delta x = 0.01 \)
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Summary

Average rate of change of \( f \) between \( a \) and \( b \) is the slope of the secant line to \( f \) between \( a \) and \( b \).

Average rate of change = \( \frac{f(b) - f(a)}{b - a} \) (also called the difference quotient)

Instantaneous rate of change is only at one point \( A \) is the same as the slope of the tangent line to the curve at \( A \).

<table>
<thead>
<tr>
<th>( t ) (years since 1978)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P ) (% with microwave)</td>
<td>8</td>
<td>14</td>
<td>21</td>
<td>34</td>
<td>61</td>
<td>79</td>
</tr>
</tbody>
</table>

6. If \( P = f(t) \) is the percentage of households with a microwave oven \( t \) years since 1978.

(a) Does \( f'(6) \) appear positive or negative?

(b) What are the units on \( f'(t) \)?

7. From the graph answer the following:

(a) At what labeled points is the derivative:
positive?  
negative?  
zero?

(b) Between which pair of consecutive points is the average rate of change:
greatest?  
least?  
closest to zero?

(c) At what point(s) is the instantaneous rate of change: greatest?  
least?  
closest to zero?

Given the graph of \( f(x) \), answer the following:

(a) Indicate whether each of the following quantities is positive or negative and illustrate your answers graphically:

i. \( f'(0) \)

ii. \( f(3) - f(2) \)

iii. \( \frac{f(5) - f(1)}{4} \)

(b) Arrange the quantities in part (a) in ascending order.

(c) Find all \( x \) values at which the derivative of \( f \) is zero.